```
(%i1)  "*****************************************************"$
(%i2)  " Implikation und Äquivalenz                          "$
(%i3)  "*****************************************************"$
(%i4)  imp(a,b):= not a or b;
(%o4)   imp(a , b) := not a   or   b
(%i5)  imp(true,true);
(%o5)   true
(%i6)  imp(true,false);
(%o6)   false
(%i7)  imp(false,true);
(%o7)   true
(%i8)  imp(false,false);
(%o8)   true
(%i9)  "*****************************************************"$
(%i10)  eqv(a,b):=imp(a,b)and imp(b,a);
(%o10)   eqv(a , b) := imp(a , b)  and   imp(b , a)
(%i11)  eqv(true,true);
(%o11)   true
(%i12)  eqv(true,false);
(%o12)   false
(%i13)  eqv(false,true);
(%o13)   false
(%i14)  eqv(false,false);
(%o14)   true
(%i15)  "*****************************************************"$
(%i16)
```

Die Negation einer Aussage a ist die Verneinung. Sie wird durch "not a" dargestellt.

Die Konjunktion von Aussagen ist die Und-Verknüpfung und wird durch "a and b" dargestellt.

Die Disjunktion von Aussagen ist die Oder-Verknüpfung und wird durch "a or b" dargestellt.

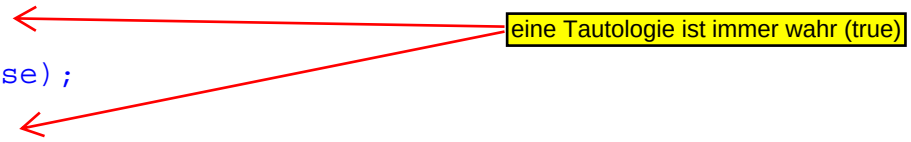
Zwei Aussagen sind äquivalent, wenn sie den gleichen Wahrheitswert haben!

Die Implikation und die Äquivalenz sind zusammengesetzte Verknüpfungen.

```
(%i1)  "*****************************************************"$
(%i2)  " TO BE OR NOT TO BE     ( Gesetz vom ausgeschlossenen Dritten ) "$
(%i3)  "*****************************************************"$
(%i9)  f(p):=p or not p;
(%o9)   f(p) := p   or   not p
(%i10)  "*****************************************************"$
(%i11)  f(true);
(%o11)   true
(%i12)  f(false);
(%o12)   true
(%i13)  "*****************************************************"$
(%i14)  " Tautologie = immer wahr                              "$
(%i15)  "*****************************************************"$
(%i16)
```

or ..... Disjunktion

not ..... Negation



Eine Tautologie von Aussagen ist eine Zusammensetzung von Aussagen, die immer wahr ist. Eine Aussage kann wahr oder falsch sein.

wahr = true
falsch = false

```
(%i1)  "*************************************************"$
(%i2)  " Die Dame, die Eurostücke ausgibt                "$
(%i3)  "*************************************************"$
(%i4)  " Eine Dame hat eine Anzahl Eurostücke in der Tasche "$
(%i5)  " und sonst kein Geld.                            "$
(%i6)  " Diese Anzahl der Eurostücke sei x               "$
(%i7)  " 1.) Die Hälfte des Geldes gibt sie für einen Hut   "$
(%i8)  "     und einen Euro spendet sie einem Bettler    "$
(%i9)  "     vor dem Geschäft.                           "$
(%i10)  "*************************************************"$
(%i11)  g1:y=x/2-1;
```

Es ist sehr zweckmäßig, Objekte, wie Gleichungen, mit einem Bezeichner zu versehen. g1 steht stellvertretend für die gesamte Gleichung.

$$(\%o11)\quad y = \frac{x}{2} - 1$$

```
(%i12)  "*************************************************"$
(%i13)  " Es verbleiben y Geldstücke.                    "$
(%i14)  " 2.) Die Hälfte der verbleibenden Summe verbraucht "$
(%i15)  "     für ihr Mittagessen im Restaurant und gibt "$
(%i16)  "     noch 2 Euro Trinkgeld.                     "$
(%i17)  "*************************************************"$
(%i18)  g2:z=y/2-2;
```

$$(\%o18)\quad z = \frac{y}{2} - 2$$

```
(%i19)  "*************************************************"$
(%i20)  " Nun verbleiben z Geldstücke.                   "$
(%i21)  " 3.) Die Hälfte von dem, was sie nun noch hat, gibt"$
(%i22)  "     sie für ein Buch aus, und ehe sie nach Hause "$
(%i23)  "     geht, nimmt sie noch einige Drinks in einer "$
(%i24)  "     Bar zu sich. Diese kosten drei Euro.        "$
(%i25)  "*************************************************"$
(%i26)  g3:w=z/2-3;
```

$$(\%o26)\quad w = \frac{z}{2} - 3$$

```
(%i27)  "*************************************************"$
(%i28)  " w = Anzahl der verbliebenen Geldstücke.        "$
(%i30)  " 4.) Nun hat sie noch einen Euro.               "$
(%i31)  "*************************************************"$
(%i32)  g4:w=1;
```

$$(\%o32)\quad w = 1$$

```
(%i33)  "*************************************************"$
```

```
(%i34)  " Wie viele Eurostücke hatte sie anfangs, wenn      "$
(%i35)  " sie niemals Geld gewechselt hat?                  "$
(%i36)  "***************************************************"$
(%i37)  solve([g1,g2,g3,g4],[x,y,z,w]);
(%o37)   [ [ x = 42 , y = 20 , z = 8 , w = 1 ] ]
(%i38)  "***************************************************"$
(%i39)  " Sie hatte 42 Euro.                                "$
(%i40)  " Quellenachweis:                                   "$
(%i41)  " Johann Weilharter, Spaß mit Algorithmen, S. 15ff  "$
(%i42)  " Braunschweig: Vieweg 1984                         "$
(%i43)  "***************************************************"$
(%i44)
```

Lösung eines linearen Gleichungssystems mit solve

Das Buch "Spaß mit Algorithmen" war ein Lehrbuch zur Programmierung mit der Programmiersprache BASIC.

B ... Beginners
A ... All Purpose
S ... Symbolic
I ... Instruction
C... Code

```
(%i1)  "*****************************************************"$
(%i2)  " Matrizenmethode                                     "$
(%i3)  "*****************************************************"$
(%i4)  g1:y=x/2-1;
```

Mit der Oberfläche WXMAXIMA ist auch die Matrizenrechnung von Maxima gut verwendbar

$$(\%o4)\quad y = \frac{x}{2} - 1$$

```
(%i5)  g2:z=y/2-2;
```

$$(\%o5)\quad z = \frac{y}{2} - 2$$

```
(%i6)  g3:w=z/2-3;
```

$$(\%o6)\quad w = \frac{z}{2} - 3$$

```
(%i7)  g4:w=1;
```

$$(\%o7)\quad w = 1$$

```
(%i8)  "*****************************************************"$
(%i9)  A:coefmatrix([g1,g2,g3,g4],[x,y,z,w]);
```

$$(\%o9)\quad \begin{bmatrix} -\frac{1}{2} & 1 & 0 & 0 \\ 0 & -\frac{1}{2} & 1 & 0 \\ 0 & 0 & -\frac{1}{2} & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Die Koeffizientenmatrix eines Gleichungssystems kann auf einfache Art und Weise ermittelt werden.

```
(%i10)  b:matrix([-1], [-2], [-3], [1]);
```

$$(\%o10)\quad \begin{bmatrix} -1 \\ -2 \\ -3 \\ 1 \end{bmatrix}$$

Mit invert(A) kann man die inverse Matrix von A bestimmen und dann durch Matrizenmultiplikation die Lösung des Gleichungssystems ermitteln.

```
(%i11)  x:invert(A).b;
```

$$(\%o11)\quad \begin{bmatrix} 42 \\ 20 \\ 8 \\ 1 \end{bmatrix}$$

```
(%i12)
```

```
(%i1)  'diff(y(x),x)-2/x*y(x)=x^2/(1+x^2);
```

$$(\%o1)\quad \frac{d}{d\,x}y(x)-\frac{2\,y(x)}{x}=\frac{x^2}{x^2+1}$$

```
(%i2)  ode2(%,y(x),x);
```

$$(\%o2)\quad y(x)=x^2\left(\operatorname{atan}(x)+\%C\right)$$

```
(%i3)
```

Man hat üblichweise kaum Möglichkeiten, Differentialgleichungen im Unterricht der Sekundarstufe II zu behandeln. Maxima würde hier einiges ermöglichen.

(%i1) `'diff(y(x),x)+x/(x^2-1)*y(x)=1/(1-x^2);`

(%o1) $$\frac{d}{d\,x}y(x)+\frac{x\,y(x)}{x^2-1}=\frac{1}{1-x^2}$$

(%i2) `ode2(%,y(x),x);`

(%o2) $$y(x)=\%e^{-\frac{\log\left(x^2-1\right)}{2}}\left(\%C-\log\left(2\sqrt{x^2-1}+2\,x\right)\right)$$

(%i3)

Differentialgleichungen werden in der Sekundarstufe II nur selten verwendet

(%i1)
```
'diff(y(x),x)+x*y(x)=x^3;
```

(%o1)
$$\frac{d}{d\,x}\,y\left(x\right)+x\,y\left(x\right)=x^3$$

(%i2)
```
ode2(%,y(x),x);
```

(%o2)
$$y\left(x\right)=\%e^{-\frac{x^2}{2}}\left(\frac{\left(2\,x^2-4\right)\,\%e^{\frac{x^2}{2}}}{2}+\%C\right)$$

(%i3)
```
ratsimp(%);
```

(%o3)
$$y\left(x\right)=\%e^{-\frac{x^2}{2}}\left(\left(x^2-2\right)\,\%e^{\frac{x^2}{2}}+\%C\right)$$

(%i4)
```
expand(%);
```

(%o4)
$$y\left(x\right)=\%C\,\%e^{-\frac{x^2}{2}}+x^2-2$$

(%i5)

```
wxMaxima 0.6.4 http://wxmaxima.sourceforge.net
Maxima 5.9.1 http://maxima.sourceforge.net
Using Lisp Kyoto Common Lisp GCL 2.6.5 (aka GCL)
Distributed under the GNU Public License. See the file COPYING.
Dedicated to the memory of William Schelter.
This is a development version of Maxima. The function bug_report()
provides bug reporting information.
```

(%i1) `'diff(y(x),x)=y(x);`

Die Lösung dieser sehr bekannten Differentialgleichung ist die Exponentialfunktion mit der Basis e (e ist die Eulersche Zahl).

(%o1) $$\frac{d}{d\,x}\,y\left(x\right)=y\left(x\right)$$

(%i2) `ode2(%,y(x),x);`

(%o2) $$y\left(x\right)=\%C\,\%e^{x}$$

(%i3)

```
(%i1)  a*'diff(x(t),t,2)+b*diff(x(t),t)+c*x(t)=0;
```

$$(\%o1)\quad a\left(\frac{d^2}{dt^2}x(t)\right)+b\left(\frac{d}{dt}x(t)\right)+c\,x(t)=0$$

```
(%i2)  ode2(%,x(t),t);
```

Is 4 a c - b² positive, negative, or zero? positive;

$$(\%o2)\quad x(t)=\%e^{-\frac{b\,t}{2\,a}}\left(\%K1\,\sin\left(\frac{\sqrt{\frac{4\,c}{a}-\frac{b^2}{a^2}}\,t}{2}\right)+\%K2\,\cos\left(\frac{\sqrt{\frac{4\,c}{a}-\frac{b^2}{a^2}}\,t}{2}\right)\right)$$

```
(%i3)  "*************************************************"$
(%i4)  " Gedämpfte harmonische Schwingung                "$
(%i5)  "*************************************************"$
(%i6)
```

Die erste Ableitung von x(t) ist die Geschwindigkeit, die zweite Ableitung nennt man Beschleunigung.

Für b = 0 erhält man eine ungedämpfte harmonische Schwingung.

```
wxMaxima 0.6.4 http://wxmaxima.sourceforge.net
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This is a development version of Maxima. The function bug_report()
provides bug reporting information.
```

(%i1)
```
'diff(y(x),x)=(y(x)^2-x^2)/(2*x*y(x));
```

(%o1)
$$\frac{d}{d\,x}\,y(x)=\frac{y(x)^2-x^2}{2\,x\,y(x)}$$

(%i2)
```
ode2(%,y(x),x);
```

(%o2)
$$-\frac{x}{y(x)^2+x^2}=\%C$$

(%i3)
```
solve(%,y(x));
```

(%o3)
$$\left[y(x)=-\sqrt{-x^2-\frac{x}{\%C}},\ y(x)=\sqrt{-x^2-\frac{x}{\%C}}\right]$$

(%i4)
```
"*************************************************"$
```

(%i5)
```
'diff(y(x),x)=(2*x^2+y(x)^2)/(3*x*y(x));
```

(%o5)
$$\frac{d}{d\,x}\,y(x)=\frac{y(x)^2+2\,x^2}{3\,x\,y(x)}$$

(%i6)
```
ode2(%,y(x),x);
```

(%o6)
$$\frac{3\,y(x)^2-3\,x^2}{2\,x^{2/3}}=\%C$$

(%i7)
```
solve(%,y(x));
```

(%o7)
$$\left[y(x)=-\frac{\sqrt{3\,x^2+2\,\%C\,x^{2/3}}}{\sqrt{3}},\ y(x)=\frac{\sqrt{3\,x^2+2\,\%C\,x^{2/3}}}{\sqrt{3}}\right]$$

(%i8)
```
"*************************************************"$
```

(%i9)
```
'diff(y(x),x)=y(x);
```

(%o9)
$$\frac{d}{d\,x}\,y(x)=y(x)$$

das ist die bekannte Differentialgleichung der Exponentialfunktion mit der Basis e (Eulersche Zahl)

(%i10)
```
ode2(%,y(x),x);
```

(%o10)
$$y(x)=\%C\ \%e^{x}$$

(%i11)
```
"*************************************************"$
```

(%i12)
```
'diff(y(x),x,2)=y(x);
```

(%o12)
$$\frac{d^2}{d\,x^2}\,y(x)=y(x)$$

(%i13)
```
ode2(%,y(x),x);
```

(%o13)
$$y(x)=\%K1\ \%e^{x}+\%K2\ \%e^{-x}$$

(%i14)
```
"*************************************************"$
```

```
(%i15)  'diff(y(x),x,2)=-y(x);
```

$$(\%o15)\quad \frac{d^2}{d\,x^2}\,y(x) = -\,y(x)$$

Das Hookesche Gesetz hat als Lösung eine ungedämpfte harmonische Schwingung.

```
(%i16)  ode2(%,y(x),x);
```

$$(\%o16)\quad y(x) = \%K1\ \sin(x) + \%K2\ \cos(x)$$

```
(%i17)  "*****************************************************"$
(%i18)
```

```
(%i4)  'diff(y(x),x)-y(x)^2*x^2=0;
```

$$(\%o4)\quad \frac{d}{d\,x}y(x) - x^2\,y(x)^2 = 0$$

```
(%i5)  ode2(%,y(x),x);
```

$$(\%o5)\quad -\frac{1}{y(x)} = \frac{x^3}{3} + \%C$$

```
(%i6)  solve(%,y(x));
```

$$(\%o6)\quad \left[\, y(x) = -\frac{3}{x^3 + 3\ \%C} \,\right]$$

```
(%i7)  "*************************************************"$
(%i8)  " Lösung Beispiel 1                                "$
(%i9)  "*************************************************"$
(%i12)  'x*diff(y(x),x)-cos(y(x))^2=0;
```

$$(\%o12)\quad x\left(\frac{d}{d\,x}y(x)\right) - \cos(y(x))^2 = 0$$

```
(%i13)  ode2(%,y(x),x);
```

$$(\%o13)\quad \tan(y(x)) = \log(x) + \%C$$

```
(%i14)  solve(%,y(x));
SOLVE is using arc-trig functions to get a solution.
Some solutions will be lost.
```

$$(\%o14)\quad \left[\, y(x) = \operatorname{atan}(\log(x) + \%C) \,\right]$$

```
(%i15)  "*************************************************"$
(%i16)  " Lösung Beispiel 2                               "$
(%i17)  "*************************************************"$
(%i20)  'diff(y(x),x)-exp(x-y(x))*x=0;
```

$$(\%o20)\quad \frac{d}{d\,x}y(x) - x\ \%e^{x - y(x)} = 0$$

```
(%i21)  ode2(%,y(x),x);
```

$$(\%o21)\quad \%e^{y(x)} + (1 - x)\ \%e^{x} = \%C$$

```
(%i22)  solve(%,y(x));
```

$$(\%o22)\quad \left[\, y(x) = \log(x\ \%e^{x} - \%e^{x} + \%C) \,\right]$$

```
(%i23)  "*************************************************"$
(%i24)  " Lösung Beispiel 3                               "$
(%i25)  "*************************************************"$
(%i26)  'diff(y(x),x)*(1-x^2)-x*y(x)=0;
```

$$(\%o26)\quad (1 - x^2)\left(\frac{d}{d\,x}y(x)\right) - x\,y(x) = 0$$

```
(%i27)  ode2(%,y(x),x);
```

```
(%o27)  y(x) = %C %e^(-log(x^2 - 1)/2)
(%i28)  "*****************************************************"$
(%i30)  " Lösung Beispiel 4                                   "$
(%i31)  "*****************************************************"$
(%i32)  'diff(y(x),x)+exp(x)*(1+y(x)^2)=0;
(%o32)  d/d x y(x) + %e^x (y(x)^2 + 1) = 0
(%i33)  ode2(%,y(x),x);
(%o33)  - atan(y(x)) = %e^x + %C
(%i34)  solve(%,y(x));
(%o34)  [ y(x) = - tan(%e^x + %C) ]
(%i35)  "*****************************************************"$
(%i36)  " Lösung Beispiel 5                                   "$
(%i37)  "*****************************************************"$
(%i38)
```

```
(%i1)  x(t)='diff(x(t),t);
```

$$(\%o1)\quad x(t)=\frac{d}{d\,t}x(t)$$

```
(%i2)  desolve(%,x(t));
```

$$(\%o2)\quad x(t)=x(0)\,\%e^{t}$$

```
(%i3)  "*****************************************************"$
(%i4)  x(t)='diff(x(t),t,2);
```

$$(\%o4)\quad x(t)=\frac{d^{2}}{d\,t^{2}}x(t)$$

```
(%i5)  desolve(%,x(t));
```

$$(\%o5)\quad x(t)=\frac{\%e^{t}\left(\left.\frac{d}{d\,t}x(t)\right|_{t=0}+x(0)\right)}{2}-\frac{\%e^{-t}\left(\left.\frac{d}{d\,t}x(t)\right|_{t=0}-x(0)\right)}{2}$$

```
(%i6)  ode2(%o4,x(t),t);
```

$$(\%o6)\quad x(t)=\%K1\,\%e^{t}+\%K2\,\%e^{-t}$$

```
(%i7)  "*****************************************************"$
(%i9)  x(t)+'diff(x(t),t)+'diff(x(t),t,2)=0;
```

$$(\%o9)\quad \frac{d^{2}}{d\,t^{2}}x(t)+\frac{d}{d\,t}x(t)+x(t)=0$$

```
(%i10)  desolve(%,x(t));
```

$$(\%o10)\quad x(t)=\%e^{-\frac{t}{2}}\left(\frac{\sin\left(\frac{\sqrt{3}\,t}{2}\right)\left(2\left(\left.\frac{d}{d\,t}x(t)\right|_{t=0}+x(0)\right)-x(0)\right)}{\sqrt{3}}+x(0)\cos\left(\frac{\sqrt{3}\,t}{2}\right)\right)$$

```
(%i11)  ode2(%o9,x(t),t);
```

$$(\%o11)\quad x(t)=\%e^{-\frac{t}{2}}\left(\%K1\,\sin\left(\frac{\sqrt{3}\,t}{2}\right)+\%K2\,\cos\left(\frac{\sqrt{3}\,t}{2}\right)\right)$$

```
(%i12)  "*****************************************************"$
(%i13)  " Elementare Differentialgleichungen                  "$
(%i14)  "*****************************************************"$
(%i15)
```

```
(%i1)  "*************************************************"$
(%i2)  " Freier Fall                                     "$
(%i3)  "*************************************************"$
(%i4)  'diff(x(t),t,2)=g;
```

$$(\%o4)\quad \frac{d^2}{d\,t^2}\,x\left(t\right) = g$$

Das ist die Differentialgleichung für den freien Fall:
Beschleunigung = Erdbeschleunigung

```
(%i5)  ode2(%,x(t),t);
```

$$(\%o5)\quad x\left(t\right) = \frac{g\,t^2}{2} + \%K2\;t + \%K1$$

```
(%i7)  "*************************************************"$
(%i8)  " K1 ist x(0) und K2 ist v(0)                     "$
(%i9)  "*************************************************"$
(%i10)
```

```
(%i1)  "*****************************************************"$
(%i2)  " DER FREIE FALL                                      "$
(%i3)  "*****************************************************"$
(%i4)  s(t):=g/2*t^2;
(%o4)   s(t) := g/2 t^2
(%i5)  "*****************************************************"$
(%i6)  " Weg-Zeit-Gesetz                                     "$
(%i7)  "*****************************************************"$
(%i8)  v(t):=diff(s(t),t);
(%o8)   v(t) := DIFF(s(t), t)
(%i9)  v(t);
(%o9)   g t
(%i10)  "*****************************************************"$
(%i11)  " Geschwindigkeits-Zeit-Gesetz                       "$
(%i12)  "*****************************************************"$
(%i13)  a(t):=diff(v(t),t);
(%o13)   a(t) := DIFF(v(t), t)
(%i14)  a(t);
(%o14)   g
(%i15)  "*****************************************************"$
(%i16)  " Die Beschleunigung ist die Erdbeschleunigung       "$
(%i17)  "*****************************************************"$
(%i18)
```

s = Weg
g = Erdbeschleunigung
t = Zeit

die erste Ableitung von s ist die Geschwindigkeit

die erste Ableitung der Geschwindigkeit ist die Beschleunigung oder

die zweite Ableitung von s ist die Beschleunigung

```
(%i1)  f(x):=a*x^2+b*x+c;
```

$$(\%o1)\quad f(x) := a\,x^2 + b\,x + c$$

```
(%i2)  ab:diff(f(x),x);
```

$$(\%o2)\quad 2\,a\,x + b$$

die Ableitung einer quadratischen Funktion ist eine lineare Funktion

```
(%i3)  "*****************************************************"$
(%i4)  f(x):=(x+a)^2;
```

$$(\%o4)\quad f(x) := (x + a)^2$$

```
(%i5)  ab:diff(f(x),x);
```

$$(\%o5)\quad 2\,(x + a)$$

```
(%i6)  "*****************************************************"$
(%i7)  f(x):=(x+a)^3;
```

$$(\%o7)\quad f(x) := (x + a)^3$$

```
(%i8)  ab:diff(f(x),x);
```

$$(\%o8)\quad 3\,(x + a)^2$$

```
(%i9)  "*****************************************************"$
(%i10)  w(t):=a*t-1/2*b*t^2;
```

$$(\%o10)\quad w(t) := a\,t - \frac{1}{2}\,b\,t^2$$

```
(%i11)  ab:diff(w(t),t);
```

$$(\%o11)\quad a - b\,t$$

```
(%i12)  "*****************************************************"$
(%i13)  f(x):=(197-34*x^2)*(7+22*x-83*x^3);
```

$$(\%o13)\quad f(x) := (197 - 34\,x^2)\,(7 + 22\,x + (-83)\,x^3)$$

```
(%i14)  ab:diff(f(x),x);
```

$$(\%o14)\quad (22 - 249\,x^2)\,(197 - 34\,x^2) - 68\,x\,(-83\,x^3 + 22\,x + 7)$$

```
(%i15)  "*****************************************************"$
(%i16)  f(x):=(2*x+3)/(3*x+2);
```

$$(\%o16)\quad f(x) := \frac{2\,x + 3}{3\,x + 2}$$

```
(%i17)  ab:diff(f(x),x);
```

$$(\%o17)\quad \frac{2}{3\,x + 2} - \frac{3\,(2\,x + 3)}{(3\,x + 2)^2}$$

```
(%i18)  "*****************************************************"$
(%i19)  f(x):=(1+x+2*x^2+3*x^3)/(1+x+2*x^2);
```

$$(\%o19)\quad f(x) := \frac{1 + x + 2\,x^2 + 3\,x^3}{1 + x + 2\,x^2}$$

```
(%i20)  ab:diff(f(x),x);
```

(%o20) $$\frac{9\,x^2+4\,x+1}{2\,x^2+x+1}-\frac{\left(4\,x+1\right)\left(3\,x^3+2\,x^2+x+1\right)}{\left(2\,x^2+x+1\right)^2}$$

(%i21)
```
"**************************************************"$
```

(%i22)
```
f(x):=(a*x+b)/(c*x+d);
```

(%o22) $$f\left(x\right):=\frac{a\,x+b}{c\,x+d}$$

(%i23)
```
ab:diff(f(x),x);
```

(%o23) $$\frac{a}{c\,x+d}-\frac{c\left(a\,x+b\right)}{\left(c\,x+d\right)^2}$$

(%i24)
```
"**************************************************"$
```

(%i25)
```
f(x):=(x^n+a)/(x^-n+b);
```

(%o25) $$f\left(x\right):=\frac{x^{n}+a}{x^{-n}+b}$$

(%i26)
```
ab:diff(f(x),x);
```

(%o26) $$\frac{n\,x^{-n-1}\left(x^{n}+a\right)}{\left(\frac{1}{x^{n}}+b\right)^2}+\frac{n\,x^{n-1}}{\frac{1}{x^{n}}+b}$$

(%i27)
```
"**************************************************"$
```

(%i28)

```
(%i1)  "*****************************************************"$
(%i4)  f(x):=x^13;
(%o4)  f(x) := x^13
(%i5)  ab:diff(f(x),x);
(%o5)  13 x^12
(%i6)  "*****************************************************"$
(%i7)  f(x):=x^(-3/2);
                          - 3
                          ---
                           2
(%o7)  f(x) := x
(%i8)  ab:diff(f(x),x);
                          3
(%o8)  - ----------
                     5 / 2
         2 x
(%i9)  "*****************************************************"$
(%i10)  f(x):=x^(2*a);
(%o10)  f(x) := x^(2 a)
(%i11)  ab:diff(f(x),x);
(%o11)  2 a x^(2 a - 1)
(%i12)  "*****************************************************"$
(%i13)  u(t):=t^2.4;
(%o13)  u(t) := t^2.3999999999999999
(%i14)  ab:diff(u(t),t);
(%o14)  2.3999999999999999 t^1.3999999999999999
(%i15)  "*****************************************************"$
(%i16)  z(u):=u^(1/3);
(%o16)  z(u) := u^(1/3)
(%i18)  ab:diff(z(u),u);
                          1
(%o18)  ----------
                     2 / 3
        3 u
(%i19)  "*****************************************************"$
(%i20)  f(x):=(x^(-5))^(1/3);
                          1/3
(%o20)  f(x) := (x^(- 5))
(%i21)  ab:diff(f(x),x);
                          5
(%o21)  - ----------
                     8 / 3
        3 x
(%i22)  "*****************************************************"$
(%i23)  u(x):=(1/x^8)^(1/5);
```

man findet die Ableitung einer Potenz, in dem man die Hochzahl um 1 vermindert und mit der alten Hochzahl multipliziert

Das ist ein bekannter Fehler der verwendeten Version von WXMAXIMA (die überflüssige und unmotivierte Anzeige von vielen Dezimalstellen)

```
(%o23)  u(x) := (1/x^8)^(1/5)
(%i24)  ab:diff(u(x),x);
(%o24)  -8/(5 x^(13/5))
(%i25)  "*****************************************************"$
(%i26)  f(x):=2*x^a;
(%o26)  f(x) := 2 x^a
(%i27)  ab:diff(f(x),x);
(%o27)  2 a x^(a - 1)
(%i28)  "*****************************************************"$
(%i29)  f(x):=(x^3)^(1/q);
(%o29)  f(x) := (x^3)^(1/q)
(%i30)  ab:diff(f(x),x);
(%o30)  3 x^(3/q - 1)/q
(%i31)  "*****************************************************"$
(%i32)  f(x):=(1/x^m)^(1/n);
(%o32)  f(x) := (1/x^m)^(1/n)
(%i33)  ab:diff(f(x),x);
(%o33)  -m/(n x (x^m)^(1/n))
(%i34)  "*****************************************************"$
(%i35)
```

```
(%i1)  f(x):=a*x^3+6;
```

$$(\%o1)\quad f(x) := a\,x^3 + 6$$

```
(%i2)  ab:diff(f(x),x);
```

$$(\%o2)\quad 3\,a\,x^2$$

```
(%i3)  "*****************************************************"$
(%i4)  f(x):=13*x^(3/2)-c;
```

$$(\%o4)\quad f(x) := 13\,x^{3/2} - c$$

```
(%i5)  ab:diff(f(x),x);
```

$$(\%o5)\quad \frac{39\sqrt{x}}{2}$$

```
(%i6)  "*****************************************************"$
(%i7)  f(x):=12*x^(1/2)+c^(1/2);
```

$$(\%o7)\quad f(x) := 12\,x^{1/2} + c^{1/2}$$

```
(%i8)  ab:diff(f(x),x);
```

$$(\%o8)\quad \frac{6}{\sqrt{x}}$$

```
(%i9)  "*****************************************************"$
(%i10)  f(x):=c^(1/2)*x^(1/2);
```

$$(\%o10)\quad f(x) := c^{1/2}\,x^{1/2}$$

```
(%i11)  ab:diff(f(x),x);
```

$$(\%o11)\quad \frac{\sqrt{c}}{2\sqrt{x}}$$

```
(%i12)  "*****************************************************"$
(%i13)  u(x):=(a*x^n-1)/c;
```

$$(\%o13)\quad u(x) := \frac{a\,x^n - 1}{c}$$

```
(%i14)  ab:diff(u(x),x);
```

$$(\%o14)\quad \frac{a\,n\,x^{n-1}}{c}$$

```
(%i15)  "*****************************************************"$
(%i18)  f(t):=1.18*t^2+22.4;
```

$$(\%o18)\quad f(t) := 1.1799999999999999\,t^2 + 22.399999999999999$$

```
(%i19)  ab:diff(f(t),t);
```

$$(\%o19)\quad 2.3599999999999999\,t$$

```
(%i20)  "*****************************************************"$
(%i23)  l(t):=l[0]*(1+0.000012*t);
```

$$(\%o23)\quad l(t) := l_0\left(1 + 1.2\,10^{-5}\,t\right)$$

```
(%i24)  ab:diff(l(t),t);
```

(%o24) $1.2\ 10^{-5}\ l_0$

(%i25) "**********"$

(%i26)

Ableitungen haben viele Anwendungen:

* in Physik und Technik
* in der Wirtschaft

```
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```

(%i1)
```
"**************************************************"$
```

(%i2)
```
" Potenzregel "$
```

(%i3)
```
"**************************************************"$
```

(%i4)
```
f(x):=x^n;
```

(%o4)
$$f(x) := x^n$$

(%i5)
```
k(x,h):=(f(x+h)-f(x))/h;
```

Diesen Ausruck nennt man Differenzenquotient

(%o5)
$$k(x , h) := \frac{f(x + h) - f(x)}{h}$$

(%i6)
```
limit(k(x,h),h,0);
```

Dieser Grenzwert heißt Differentialquotient

(%o6)
$$n\ x^{n - 1}$$

(%i7)
```
"**************************************************"$
```

(%i8)
```
" Das ist die Regel für die Ableitung einer Potenz "$
```

(%i9)
```
"**************************************************"$
```

(%i10)

Man bestimmt die Ableitung einer Potenz, in dem man die Hochzahl um 1 vermindert und mit der alten Hochzahl multipliziert.

```
(%i1)  NB:x+y=10;
```

Die Zahl 10 soll in zwei Summanden zerlegt werden.

```
(%o1)   y + x = 10
(%i2)  NB:NB-x;
(%o2)   y = 10 - x
(%i3)  "*****************************************************"$
(%i4)  HB:x^2+y^2;
```

Die Summe der Quadrate dieser Summanden soll ein Minimum werden.

```
(%o4)   y^2 + x^2
(%i7)  HB,y=10-x;
(%o7)   x^2 + (10 - x)^2
(%i8)  "*****************************************************"$
(%i10)  f(x):=x^2+(10-x)^2;
(%o10)   f(x) := x^2 + (10 - x)^2
(%i11)  ab:diff(f(x),x);
(%o11)   2 x - 2 (10 - x)
(%i12)  solve(ab=0,x);
```

Nullsetzen der ersten Ableitung, Die horizontale Tangente als notwendige Bedingung.

```
(%o12)   [ x = 5 ]
(%i14)  y,x=5;
(%o14)   5
(%i15)  "*****************************************************"$
(%i16)  f(5);
(%o16)   50
(%i18)  ab2:diff(f(x),x,2);
(%o18)   4
```

Wenn die zweite Ableitung an der kritische Stelle > 0 ist, liegt ein Minimum vor.

```
(%i19)
```

```
(%i1)  "*****************************************************"$
(%i2)  " EINE KURVENDISKUSSION                                "$
(%i3)  "*****************************************************"$
(%i4)  f(x):=1/3*x^3-2*x^2+3*x+1;
```

$$(\%o4)\quad f(x) := \frac{1}{3}x^3 - 2x^2 + 3x + 1$$

```
(%i5)  solve(f(x)=0,x);
```

Bestimmung der Nullstellen (Schnittpunkte mit der x-Achse)

$$(\%o5)\quad \left[x = \frac{\frac{\sqrt{3}\,\%i}{2} - \frac{1}{2}}{\left(\frac{\sqrt{21}}{2} - \frac{5}{2}\right)^{1/3}} + \left(\frac{\sqrt{21}}{2} - \frac{5}{2}\right)^{1/3}\left(-\frac{\sqrt{3}\,\%i}{2} - \frac{1}{2}\right) + 2\ ,\ x = \left(\frac{\sqrt{21}}{2} - \frac{5}{2}\right)^{1/3}\left(\frac{\sqrt{3}\,\%i}{2} - \frac{1}{2}\right) + \frac{-\frac{\sqrt{3}\,\%i}{2} - \frac{1}{2}}{\left(\frac{\sqrt{21}}{2} - \frac{5}{2}\right)^{1/3}} + 2\ ,\ x = \left(\frac{\sqrt{21}}{2} - \frac{5}{2}\right)^{1/3} + \frac{1}{\left(\frac{\sqrt{21}}{2} - \frac{5}{2}\right)^{1/3}} + 2\right]$$

```
(%i6)  %,numer;
(%o6)  [x = - 1.6858441411316549 (0.8660254037844386 %i - 0.5) -
0.59317464503493833 (- 0.8660254037844386 %i - 0.5) + 2 , x = -
0.59317464503493833 (0.8660254037844386 %i - 0.5) - 1.6858441411316549
(- 0.8660254037844386 %i - 0.5) + 2 , x = - 0.27901878616659326 ]
(%i7)  "*****************************************************"$
(%i8)  " Es gibt eine Nullstelle                              "$
(%i9)  "*****************************************************"$
(%i10) ab:diff(f(x),x);
```

Bestimmung der Extremwerte

$$(\%o10)\quad x^2 - 4x + 3$$

```
(%i11) solve(ab=0,x);
```

notwendige Bedingung: die erste Ableitung muss NULL sein

```
(%o11)  [x = 3 , x = 1 ]
(%i12) f(3);
(%o12)  1
(%i13) f(1);
```

$$(\%o13)\quad \frac{7}{3}$$

```
(%i14) ab2:diff(f(x),x,2);
(%o14)  2 x - 4
(%i15) ab2,x=3;
(%o15)  2
(%i16) ab2,x=1;
(%o16)  - 2
(%i17) "*****************************************************"$
(%i18) " MIN(3,1) und MAX(1,7/3)                              "$
```

```
(%i19)  "*****************************************************"$
(%i20)  solve(ab2=0,x);
(%o20)   [ x = 2 ]
(%i21)  f(2);
(%o21)  5/3
(%i22)  "*****************************************************"$
(%i23)  " WP(2,5/3)                                           "$
(%i24)  "*****************************************************"$
(%i25)
```

Bestimmung von Wendepunkten

Bei einer Kurvendiskussion bestimmt man mindestens:

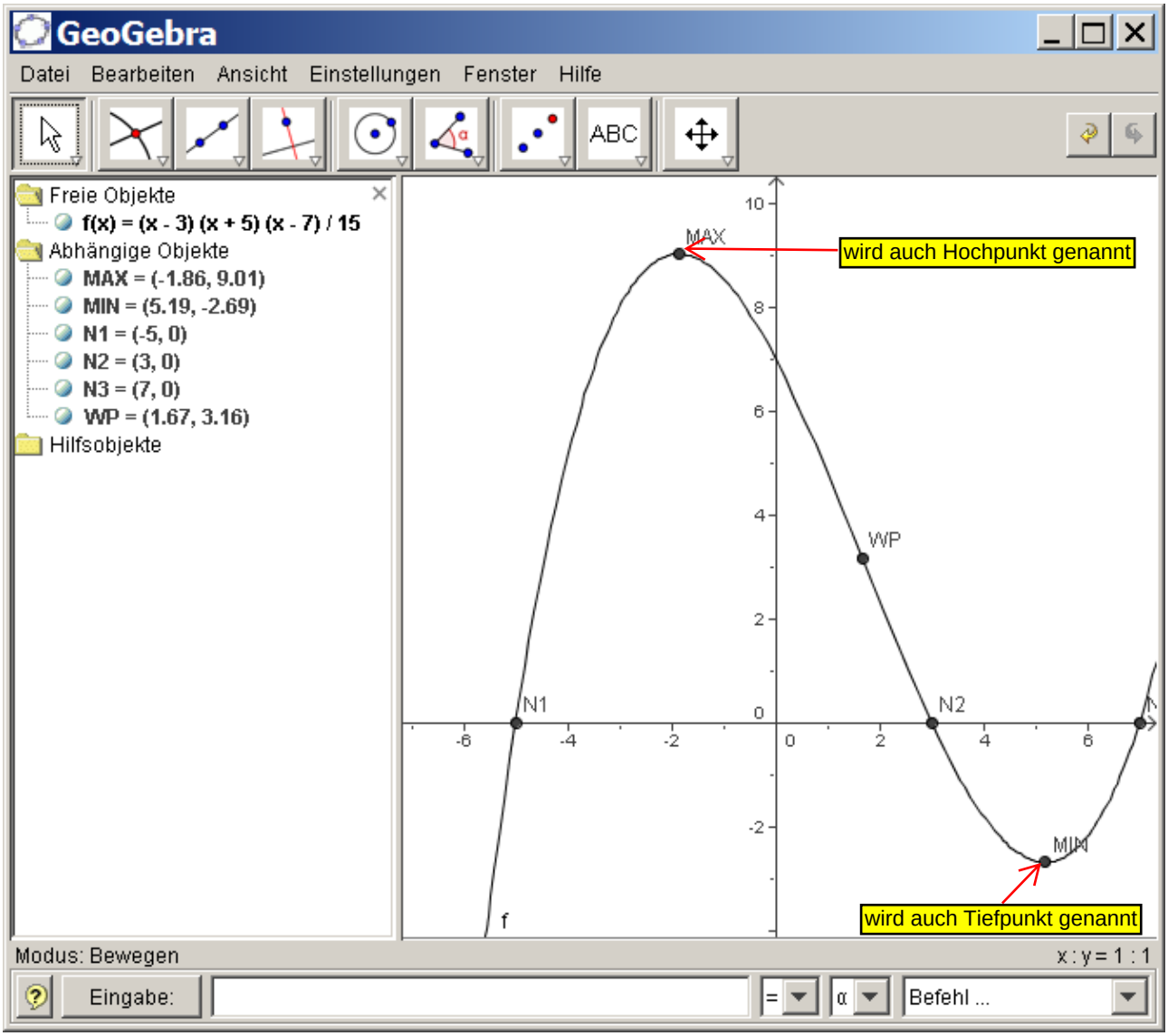
1.) Nullstellen

2.) Extremwerte

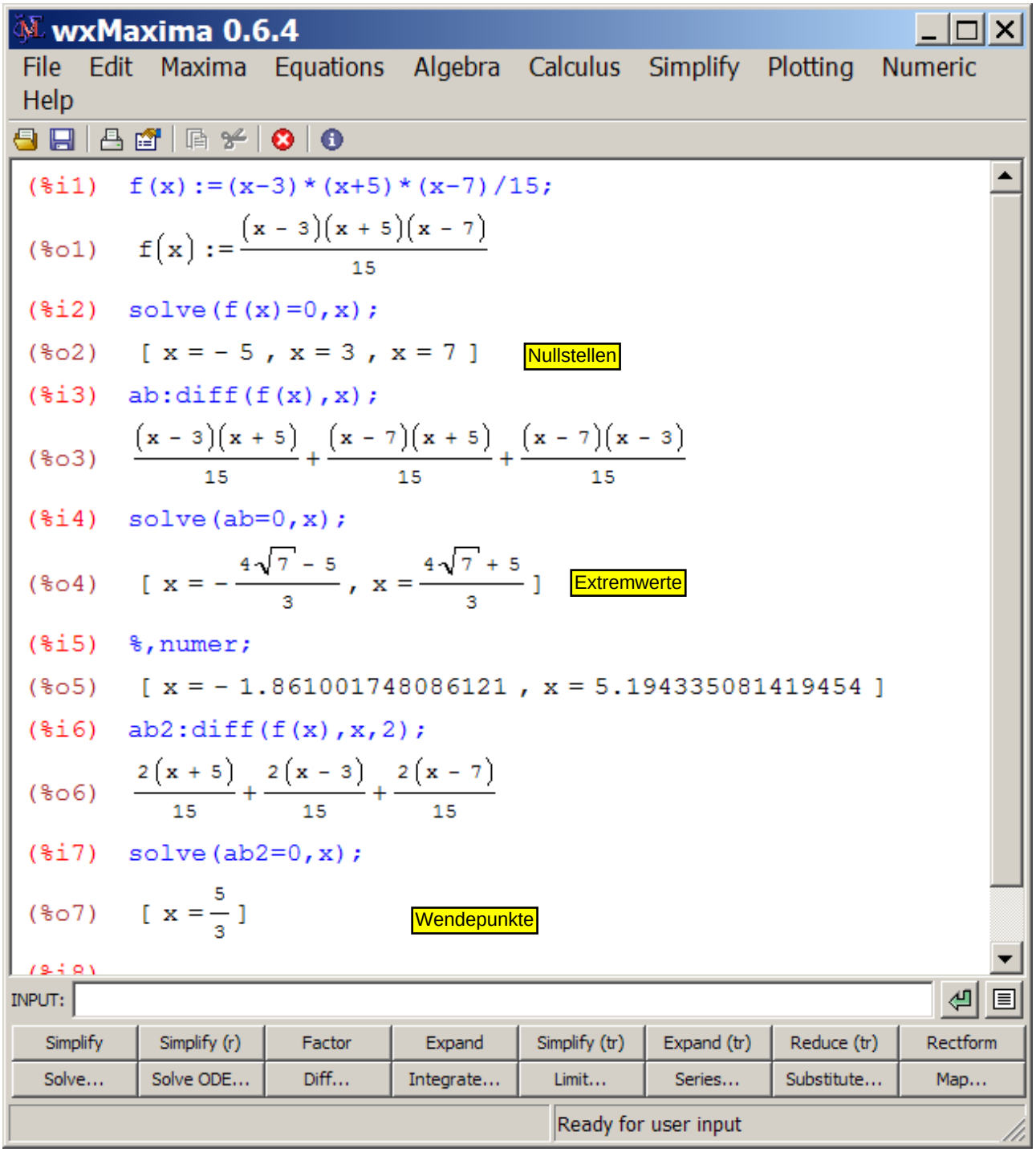
3.) Wendepunkte

Johann Weilharter, Kontrolle der Kurvendiskussion mit Maxima, Seite 1 von 2

Kontrolle der Kurvendiskussion



Wir kontrollieren diese Kurvendiskussion mit Maxima:



```
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provides bug reporting information.
(%i1)  "**************************************************"$
(%i2)  " Die merkwürdigste Gleichung "$
(%i3)  "**************************************************"$
(%i4)  %e^(%i*%pi);
(%o4)   - 1
(%i5)  "**************************************************"$
(%i6)  " %e ist die Eulersche Zahl "$
(%i7)  " %i ist die imaginäre Einheit "$
(%i8)  " %pi ist die Kreiszahl "$
(%i9)  "**************************************************"$
(%i10)
```
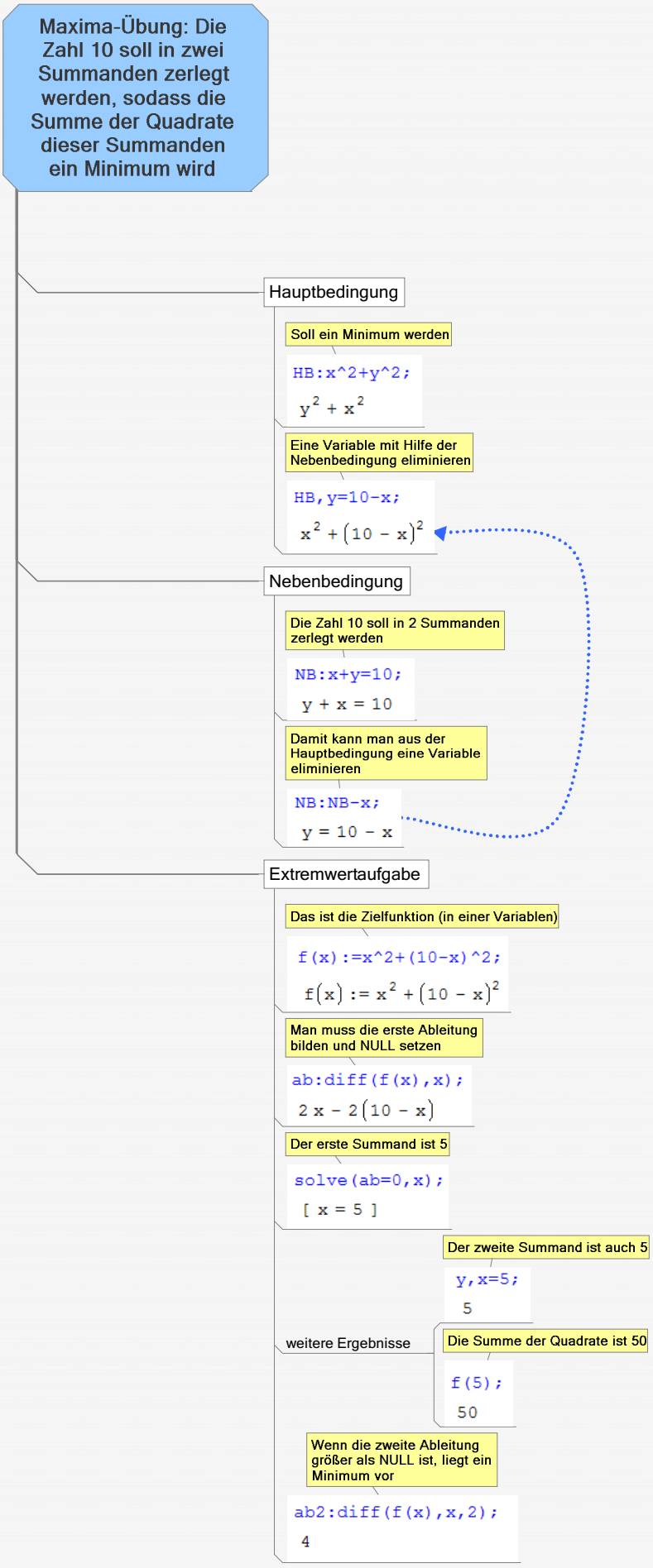

Analysis: eine sehr bekannte Extremwertaufgabe, die schon weiter vorn beschrieben wurde



```
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```

```
(%i1)  "*****************************************************************"$
(%i2)  " Kosten- und Preistheorie "$
(%i3)  "*****************************************************************"$
(%i4)  K(x):=x^2+8*x+25;
```

$$(\%o4)\quad K(x) := x^2 + 8\,x + 25$$

Gesamtkostenfunktion

```
(%i5)  "*****************************************************************"$
(%i7)  " Gegeben ist also eine quadratische Kostenfunktion "$
(%i8)  "*****************************************************************"$
(%i9)  DK(x):=K(x)/x;
```

Durchschnittskostenfunktion

$$(\%o9)\quad DK(x) := \frac{K(x)}{x}$$

```
(%i10)
"*****************************************************************"$
(%i11)  " Durchschnittskosten = Stückkosten "$
(%i12)
"*****************************************************************"$
(%i13)  ab:diff(DK(x),x);
```

Ableitung der Durchschnittskosten

$$(\%o13)\quad \frac{2\,x + 8}{x} - \frac{x^2 + 8\,x + 25}{x^2}$$

```
(%i14)  solve(ab=0,x);
```

$$(\%o14)\quad [\,x = -5\,,\ x = 5\,]$$

das Betriebsoptimum ist jene Produktionsmenge, bei der die Durchschnittskosten am kleinsten sind

```
(%i15)
"*****************************************************************"$
(%i16)  " Das Betriebsoptimum ist x=5 "$
(%i17)
"*****************************************************************"$
(%i18)  DK(5);
```

$$(\%o18)\quad 18$$

das ist die langfristige Preisuntergrenze (das Minimum der Durchschnittskosten)

```
(%i19)
"*****************************************************************"$
(%i20)  " Das Minimum der Durchschnittskosten ist 18 "$
(%i21)
"*****************************************************************"$
```

```
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```

(%i1)
```
"*****************************************************"$
```

(%i2)
```
" Betriebsoptimum: Kostenfunktion dritten Grades "$
```

(%i3)
```
"*****************************************************"$
```

(%i4)
```
K(x):=0.001*x^3+0.5*x^2+10*x+1000;
```

(%o4)
$$K(x) := 0.001\,x^3 + 0.5\,x^2 + 10\,x + 1000$$

(%i5)
```
DK(x):=K(x)/x;
```

(%o5)
$$DK(x) := \frac{K(x)}{x}$$

der bekannte Anzeigefehler der verwendeten WXMAXIMA-Version

(%i6)
```
ab:diff(DK(x),x);
```

(%o6)
$$\frac{0.0030000000000000001\,x^2 + x + 10}{x} - \frac{0.001\,x^3 + 0.5\,x^2 + 10\,x + 1000}{x^2}$$

(%i7)
```
solve(ab=0,x);
```

```
RAT replaced 0.003 by 3//1000 = 0.003
RAT replaced 0.5 by 1//2 = 0.5
RAT replaced 0.001 by 1//1000 = 0.001
```

(%o7)
$$[\,x = -100\sqrt{2} - 100\,,\ x = 100\sqrt{2} - 100\,,\ x = -50\,]$$

(%i8)
```
%,numer;
```

(%o8)
$$[\,x = -241.42135623730951\,,\ x = 41.42135623730951\,,\ x = -50\,]$$

(%i9)
```
"*****************************************************"$
```

(%i10)
```
" Das Betriebsoptimum ist 41 "$
```

die anderen Werte sind unzulässig (und theoretisch Stellen eines Maximums)

(%i11)
```
"*****************************************************"$
```

(%i12)
```
DK(41);
```

(%o12)
```
56.571243902439029
```

(%i13)
```
"*****************************************************"$
```

(%i14)
```
" Die langfristige Preisuntergrenze ist 56,57 "$
```

(%i15)
```
"*****************************************************"$
```

(%i16)

die langfristige Preisuntergrenze ist identisch mit dem Minimum der Stückkosten (Durchschnittskosten)

```
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```

```
(%i1)  "*****************************************************"$
(%i2)  " Grenzkosten = Durchschnittskosten (Betriebsoptimum)"$
(%i3)  "*****************************************************"$
(%i4)  K(x):=x^2+8*x+36;
```

$$(\%o4)\quad K(x) := x^2 + 8\,x + 36$$

```
(%i5)  GK(x):=diff(K(x),x);
```

$$(\%o5)\quad GK(x) := \mathrm{DIFF}(K(x),\, x)$$

```
(%i6)  "*****************************************************"$
(%i7)  " Grenzkosten = Ableitung der Kosten "$
(%i8)  "*****************************************************"$
(%i9)  DK(x):=K(x)/x;
```

$$(\%o9)\quad DK(x) := \frac{K(x)}{x}$$

Im Betriebsoptimum schneiden sich Grenzkosten und Durchschnittskosten

```
(%i10)  g:GK(x)=DK(x);
```

$$(\%o10)\quad 2\,x + 8 = \frac{x^2 + 8\,x + 36}{x}$$

```
(%i11)  solve(g,x);
```

$$(\%o11)\quad [\,x = -6\,,\ x = 6\,]$$

```
(%i12)  "*****************************************************"$
(%i13)  " Das Betriebsoptimum ist x=6 "$
(%i14)  "*****************************************************"$
(%i15)
```

```
(%i1)  "*****************************************************"$
(%i2)  " Quadratische Nachfragefunktion:                     "$
(%i3)  " Für einen Preis von p=5 € wird ein Absatz von 1000 "$
(%i4)  " Stück erwartet. Die Sättigungsmenge ist 5000 Stück."$
(%i5)  " Die Preisobergrenze ist 10 €                        "$
(%i6)  " Bestimme eine quadratische Nachfragefunktion!       "$
(%i7)  "*****************************************************"$
(%i8)  x1:1000;
(%o8)   1000
(%i9)  p1:5;
(%o9)   5
(%i10)  x2:5000;
(%o10)   5000
(%i11)  p2:0;
(%o11)   0
(%i12)  x3:0;
(%o12)   0
(%i13)  p3:10;
(%o13)   10
(%i14)  "*****************************************************"$
(%i15)  g(x,p):=p=a*x^2+b*x+c;
(%o15)   g(x , p) := p = a x^2 + b x + c
(%i16)  g1:g(x1,p1);
(%o16)   5 = c + 1000 b + 1000000 a
(%i17)  g2:g(x2,p2);
(%o17)   0 = c + 5000 b + 25000000 a
(%i18)  g3:g(x3,p3);
(%o18)   10 = c
(%i19)  "*****************************************************"$
(%i20)  solve([g1,g2,g3],[a,b,c]);
(%o20)   [ [ a = 3/4000000 , b = - 23/4000 , c = 10 ] ]
(%i21)  "*****************************************************"$
(%i22)  p(x):=3/4000000*x^2-23/4000*x+10;
(%o22)   p(x) := 3/4000000 x^2 - 23/4000 x + 10
(%i23)  "*****************************************************"$
(%i24)  U(x):=p(x)*x;
(%o24)   U(x) := p(x) x
```

Koordinaten des ersten Punktes

Koordinaten des zweiten Punktes

Koordinaten des dritten Punktes

Ansatz. quadratische Funktion

Punkte einsetzen

Gleichungssystem lösen

die gefundene Nachfragefunktion

```
(%i26)  ab:diff(U(x),x);
```

$$(\%o26)\quad \frac{3\,x^2}{4000000}+\left(\frac{3\,x}{2000000}-\frac{23}{4000}\right)x-\frac{23\,x}{4000}+10$$

```
(%i27)  solve(ab=0,x);
```

$$(\%o27)\quad [\,x=\frac{10000}{9},\ x=4000\,]$$

```
(%i29)  ab2:diff(U(x),x,2);
```

$$(\%o29)\quad \frac{9\,x}{2000000}-\frac{23}{2000}$$

```
(%i30)  ab2,x=4000;
```

$$(\%o30)\quad \frac{13}{2000}$$

```
(%i31)  ab2,x=10000/9;
```

das ist die umsatzmaximale Menge, weil die zweite Ableitung < 0 ist

$$(\%o31)\quad -\frac{13}{2000}$$

```
(%i32)  p(10000/9);
```

$$(\%o32)\quad \frac{245}{54}$$

das ist der umsatzmaximale Preis

```
(%i33)  %,numer;
(%o33)  4.5370370370370372
(%i34)  "*************************************************"$
(%i35)  " Der optimale Preis ist 4,54 €                   "$
(%i36)  "*************************************************"$
(%i37)  U(10000/9);
```

$$(\%o37)\quad \frac{1225000}{243}$$

das ist der maximale Umsatz

```
(%i38)  %,numer;
(%o38)  5041.1522633744853
(%i39)  "*************************************************"$
(%i40)  " Der optimale Umsatz ist 5.041,15                "$
(%i41)  "*************************************************"$
(%i42)  U(4000);
(%o42)  - 4000
(%i43)  solve(p(x)=0,x);
```

$$(\%o43)\quad [\,x=\frac{8000}{3},\ x=5000\,]$$

```
(%i44)  %,numer;
(%o44)  [ x = 2666.6666666666665 , x = 5000 ]
(%i47)  " DIESER BEREICH HAT KEINEN PRAKTISCHEN SINN      "$
(%i48)  "=================================================="$
```

```
(%i1)  "*****************************************************"$
(%i2)  " Kosten- und Preistheorie                             "$
(%i3)  "*****************************************************"$
(%i4)  K(x):=1/1000*x^3+1/2*x^2+10*x+1000;
```

$$(\%o4)\quad K(x) := \frac{1}{1000}\,x^3 + \frac{1}{2}\,x^2 + 10\,x + 1000$$

```
(%i5)  "*****************************************************"$
(%i6)  " Berechnung der Grenzkosten                           "$
(%i7)  "*****************************************************"$
(%i8)  GK:diff(K(x),x);
```

$$(\%o8)\quad \frac{3\,x^2}{1000} + x + 10$$

```
(%i9)  "*****************************************************"$
(%i10)  " Berechnung der Stückkosten                           "$
(%i11)  "*****************************************************"$
(%i12)  DK(x):=K(x)/x;
```

$$(\%o12)\quad DK(x) := \frac{K(x)}{x}$$

Stückkosten = Gesamtkosten / Menge

```
(%i13)  DK(x);
```

$$(\%o13)\quad \frac{\frac{x^3}{1000} + \frac{x^2}{2} + 10\,x + 1000}{x}$$

```
(%i14)  "*****************************************************"$
(%i15)  g:GK=DK(x);
```

die Lösung dieser Gleichung ist das Betriebsoptimum

$$(\%o15)\quad \frac{3\,x^2}{1000} + x + 10 = \frac{\frac{x^3}{1000} + \frac{x^2}{2} + 10\,x + 1000}{x}$$

```
(%i16)  solve(g,x);
```

$$(\%o16)\quad [\,x = -100\sqrt{2} - 100\,,\ x = 100\sqrt{2} - 100\,,\ x = -50\,]$$

```
(%i17)  "*****************************************************"$
(%i18)  ab:diff(DK(x),x);
```

$$(\%o18)\quad \frac{\frac{3\,x^2}{1000} + x + 10}{x} - \frac{\frac{x^3}{1000} + \frac{x^2}{2} + 10\,x + 1000}{x^2}$$

```
(%i19)  solve(ab=0,x);
```

$$(\%o19)\quad [\,x = -100\sqrt{2} - 100\,,\ x = 100\sqrt{2} - 100\,,\ x = -50\,]$$

```
(%i20)  "*****************************************************"$
(%i21)  " Das Betriebsoptimum ist ungefähr 41 (warum?)        "$
(%i22)  "*****************************************************"$
(%i23)
```

```
(%i1)  "*************************************************"$
(%i2)  " Zinsenrechnung - Listenverarbeitung             "$
(%i3)  "*************************************************"$
(%i4)  K:[1000,2000,3000,4000,5000];
(%o4)   [ 1000 , 2000 , 3000 , 4000 , 5000 ]
(%i5)  p:[2,3,4,5,6];
(%o5)   [ 2 , 3 , 4 , 5 , 6 ]
(%i6)  t:[180,360,270,180,90];
(%o6)   [ 180 , 360 , 270 , 180 , 90 ]
(%i7)  "*************************************************"$
(%i8)  Z:K*p*t/36000;
(%o8)   [ 10 , 60 , 90 , 100 , 75 ]
(%i9)  "*************************************************"$
(%i10)  " Wie hoch ist die Zinssumme?                    "$
(%i11)  "*************************************************"$
(%i12)  Zinssumme:sum(Z[k],k,1,5);
(%o12)   335
(%i13)  "*************************************************"$
(%i14)  " Wie hoch ist die eingesetzte Kapitalsumme?     "$
(%i15)  "*************************************************"$
(%i16)  Kapitalsumme:sum(K[k],k,1,5);
(%o16)   15000
(%i17)  "*************************************************"$
(%i18)  " Wie hoch sind die Endkapitalien?               "$
(%i19)  "*************************************************"$
(%i20)  EK:K+Z;
(%o20)   [ 1010 , 2060 , 3090 , 4100 , 5075 ]
(%i21)  "*************************************************"$
(%i22)  " Wie hoch ist die Endkapitalsumme?              "$
(%i23)  "*************************************************"$
(%i24)  Endkapitalsumme:sum(EK[i],i,1,5);
(%o24)   15335
(%i25)  Kapitalsumme+Zinssumme;
(%o25)   15335
(%i26)  "*************************************************"$
(%i27)  " Das ist quasi die Probe                        "$
(%i28)  "*************************************************"$
(%i29)
```

5 Kapitalien

die zugehörigen Zinssätze

die Verzinsungsdauern

die Zinsen mit Listenverarbeitung ermittelt

eine schöne Anwendung der Listenverarbeitung

Verwendung von indizierten Variablen

```
(%i1)  "*****************************************************"$
(%i2)  " Arithmetische und geometrische Folgen                "$
(%i3)  "*****************************************************"$
(%i4)  f(n):=a[n+1]:a[n]+d;
```

arithmetische Folge: der Abstand zweier benachbarter Folgenglieder ist konstant

$$(\%o4)\quad f(n) := a_{n+1} : a_n + d$$

```
(%i5)  for n:1 thru 5 do display(f(n),a[n]);
```

$$f(1) = d + a_1\ a_1 = a_1\ f(2) = 2\ d + a_1\ a_2 = d + a_1\ f(3) = 3\ d + a_1\ a_3 = 2\ d + a_1\ f(4) = 4\ d + a_1\ a_4 = 3\ d + a_1\ f(5) = 5\ d + a_1\ a_5 = 4\ d + a_1$$

```
(%o5)  DONE
(%i6)  "*****************************************************"$
(%i7)  f(n):=b[n+1]:b[n]*q;
```

geometrische Folge: das Verhältnis zweiter benachbarter Folgenglieder ist konstant

$$(\%o7)\quad f(n) := b_{n+1} : b_n\ q$$

```
(%i8)  for n:1 thru 5 do display(f(n),b[n]);
```

$$f(1) = b_1\ q\ b_1 = b_1\ f(2) = b_1\ q^2\ b_2 = b_1\ q\ f(3) = b_1\ q^3\ b_3 = b_1\ q^2\ f(4) = b_1\ q^4\ b_4 = b_1\ q^3\ f(5) = b_1\ q^5\ b_5 = b_1\ q^4$$

```
(%o8)  DONE
(%i9)  "*****************************************************"$
(%i10)
```

```
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```

```
(%i1)  "*****************************************************"$
(%i2)  " Grenzwerte von Funktionen "$
(%i3)  "*****************************************************"$
(%i4)  f(x):=(exp(x^2/2)-1)/x^2;
```

$$(\%o4)\quad f(x) := \frac{\mathrm{EXP}\left(\frac{x^2}{2}\right) - 1}{x^2}$$

```
(%i5)  limit(f(x),x,0);
```

limit = limes = Grenzwert

$$(\%o5)\quad \frac{1}{2}$$

```
(%i6)  "*****************************************************"$
(%i7)  f(x):=(exp(x^2)-1)/x^2;
```

$$(\%o7)\quad f(x) := \frac{\mathrm{EXP}\left(x^2\right) - 1}{x^2}$$

```
(%i8)  limit(f(x),x,0);
(%o8)  1
(%i9)  "*****************************************************"$
(%i10)  f(x):=(sin(2*x)/x)^5;
```

$$(\%o10)\quad f(x) := \left(\frac{\sin(2\ x)}{x}\right)^5$$

```
(%i11)  limit(f(x),x,0);
(%o11)  32
(%i12)  "*****************************************************"$
(%i13)  f(x):=log(1+4*x+x^2)/x;
```

$$(\%o13)\quad f(x) := \frac{\log\left(1 + 4\ x + x^2\right)}{x}$$

```
(%i14)  limit(f(x),x,0);
(%o14)  4
(%i15)  "*****************************************************"$
(%i16)
```

```
(%i1)  "*****************************************************"$
(%i2)  " Lineare Abschreibung                                 "$
(%i3)  "*****************************************************"$
(%i4)  " B ... Buchwert nach n Jahren                         "$
(%i5)  " A ... Anschaffungswert                               "$
(%i6)  " n ... Nutzungsdauer                                  "$
(%i7)  " x ... Anzahl der Jahre                               "$
(%i8)  "*****************************************************"$
(%i9)  B(x):=A-A/n*x;
```

$$(\%o9)\quad B\left(x\right) := A - \frac{A}{n}\,x$$

Funktion der linearen Abschreibung

```
(%i10)  "*****************************************************"$
(%i14)  Abschreibung:B=A-A/n*x;
```

$$(\%o14)\quad B = A - \frac{A\,x}{n}$$

```
(%i15)  "*****************************************************"$
(%i16)  solve(Abschreibung,A);
```

$$(\%o16)\quad \left[\, A = -\frac{B\,n}{x - n} \,\right]$$

```
(%i17)  "*****************************************************"$
(%i18)  " Formel für den Anschaffungswert                     "$
(%i19)  "*****************************************************"$
(%i20)  solve(Abschreibung,n);
```

$$(\%o20)\quad \left[\, n = -\frac{A\,x}{B - A} \,\right]$$

```
(%i21)  "*****************************************************"$
(%i22)  " Formel für die Nutzungsdauer                        "$
(%i23)  "*****************************************************"$
(%i24)  solve(Abschreibung,x);
```

$$(\%o24)\quad \left[\, x = -\frac{\left(B - A\right) n}{A} \,\right]$$

```
(%i25)  "*****************************************************"$
(%i26)  " Formel für die Anzahl der Jahre                     "$
(%i27)  "*****************************************************"$
(%i28)
```

```
(%i1)  "*****************************************************"$
(%i2)  " Bestimme die Schnittpunkte der Polynomfunktionen   "$
(%i3)  "*****************************************************"$
(%i4)  f(x):=2*x^2-3*x+2;
(%o4)  f(x) := 2 x^2 - 3 x + 2
(%i5)  g(x):=x^2-4*x+4;
(%o5)  g(x) := x^2 - 4 x + 4
(%i6)  solve(f(x)=g(x),x);
(%o6)  [ x = 1 , x = - 2 ]
(%i7)  f(1);
(%o7)  1
(%i8)  f(-2);
(%o8)  16
(%i9)  "*****************************************************"$
(%i10)  " P1(1,1), P2(-2,16)                                 "$
(%i11)  "*****************************************************"$
(%i12)  f(x):=x^2-4*x+3;
(%o12)  f(x) := x^2 - 4 x + 3
(%i13)  g(x):=-x^2+x+1;
(%o13)  g(x) := - x^2 + x + 1
(%i14)  solve(f(x)=g(x),x);
(%o14)  [ x = 2 , x = 1/2 ]
(%i15)  f(2);
(%o15)  - 1
(%i16)  f(1/2);
(%o16)  5/4
(%i17)  "*****************************************************"$
(%i18)  " Auch hier zwei Punkte: P(2,-1) und Q(1/2,5/4)       "$
(%i19)  "*****************************************************"$
(%i20)  f(x):=x^2+6*x+9;
(%o20)  f(x) := x^2 + 6 x + 9
(%i21)  g(x):=-x^2+5*x+10;
(%o21)  g(x) := - x^2 + 5 x + 10
(%i22)  solve(f(x)=g(x),x);
(%o22)  [ x = 1/2 , x = - 1 ]
```

erste Polynomfunktion

zweite Polynomfunktion

```
(%i23)  f(1/2);
(%o23)  49/4
(%i24)  f(-1);
(%o24)  4
(%i25)  "*****************************************************"$
(%i26)  " P(1/2,49/4) , Q(-1,4)                               "$
(%i27)  "*****************************************************"$
(%i28)  f(x):=x^3-x^2+2*x-2;
(%o28)  f(x) := x^3 - x^2 + 2 x - 2
(%i29)  g(x):=x^3-2*x^2+3*x;
(%o29)  g(x) := x^3 - 2 x^2 + 3 x
(%i30)  solve(f(x)=g(x),x);
(%o30)  [ x = 2 , x = - 1 ]
(%i31)  f(2);
(%o31)  6
(%i32)  f(-1);
(%o32)  - 6
(%i33)  "*****************************************************"$
(%i34)  " da x^3 aus der Rechnung fällt, gibt es nur 2        "$
(%i36)  " Schnittpunkte                                       "$
(%i37)  "*****************************************************"$
(%i38)
```

```
(%i1)  "*************************************************"$
(%i2)  " Bestimmen Sie die Polynomfunktionen             "$
(%i3)  "*************************************************"$
(%i4)  " Polynomfunktion zweiten Grades, deren Graph durch  "$
(%i5)  " die Punkte A(-2,3) und B(-1,1.5) verläuft und die  "$
(%i6)  " y-Achse bei y=1 schneidet                       "$
(%i7)  "*************************************************"$
(%i8)  g(x,y):=y=a*x^2+b*x+c;
(%o8)  g(x , y) := y = a x^2 + b x + c
(%i9)  x1:-2;
(%o9)  - 2
(%i10)  y1:3;
(%o10)  3
(%i11)  x2:-1;
(%o11)  - 1
(%i12)  y2:1.5;
(%o12)  1.5
(%i13)  x3:0;
(%o13)  0
(%i14)  y3:1;
(%o14)  1
(%i15)  g1:g(x1,y1);
(%o15)  3 = c - 2 b + 4 a
(%i16)  g2:g(x2,y2);
(%o16)  1.5 = c - b + a
(%i17)  g3:g(x3,y3);
(%o17)  1 = c
(%i18)  solve([g1,g2,g3],[a,b,c]);
RAT replaced 1.5 by 3//2 = 1.5
(%o18)  [ [ a = 1/2 , b = 0 , c = 1 ] ]
(%i19)  y=1/2*x^2+1;
(%o19)  y = x^2/2 + 1
(%i20)  "*************************************************"$
(%i21)  " Polynomfunktion zweiten Grades, deren Graph    "$
(%i22)  " durch die Punkte A(1,1) und B(2,4) verläuft    "$
(%i23)  " und die y-Achse bei y=2 schneidet              "$
(%i24)  "*************************************************"$
```

solche Aufgaben behandelt man unter dem Thema "umgekehrte Kurvendiskussion"

die Auflösung eines Gleichungssystems ist erforderlich

das ist die gesuchte Funktion

```
(%i26)  x1:1;
(%o26)   1
(%i27)  y1:1;
(%o27)   1
(%i28)  x2:2;
(%o28)   2
(%i29)  y2:4;
(%o29)   4
(%i30)  x3:0;
(%o30)   0
(%i31)  y3:1;
(%o31)   1
(%i37)  g(x,y):=y=a*x^2+b*x+c;
```

Ansatz der Gleichung als Funktion in zwei Variablen macht das Einsetzen von Punkten ganz besonders einfach.

$$(\%o37)\quad g(x,y) := y = a\,x^2 + b\,x + c$$

```
(%i38)  g1:g(x1,y1);
```

$$(\%o38)\quad 1 = c + b + a$$

```
(%i39)  g2:g(x2,y2);
```

$$(\%o39)\quad 4 = c + 2\,b + 4\,a$$

```
(%i40)  g3:g(x3,y3);
```

$$(\%o40)\quad 1 = c$$

```
(%i41)  solve([g1,g2,g3],[a,b,c]);
```

die Lösung des Gleichungssystems

$$(\%o41)\quad \left[\left[a = \frac{3}{2},\ b = -\frac{3}{2},\ c = 1\right]\right]$$

```
(%i42)  y=3/2*x^2-3/2*x+1;
```

$$(\%o42)\quad y = \frac{3\,x^2}{2} - \frac{3\,x}{2} + 1$$

```
(%i43)  "*************************************************"$
(%i44)
```

```
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This is a development version of Maxima. The function bug_report()
provides bug reporting information.
(%i1)  "*****************************************************"$
(%i2)  " Variable in einem Term mit Werten belegen: "$
(%i3)  "*****************************************************"$
(%i4)  T1:(x+1)^3;
(%o4)  (x + 1)^3
(%i5)  "*****************************************************"$
(%i6)  T1,x=0;
(%o6)  1
(%i7)  "*****************************************************"$
(%i8)  T1,x=1;
(%o8)  8
(%i9)  "*****************************************************"$
(%i10)  T1,x=2;
(%o10)  27
(%i11)  "*****************************************************"$
(%i12)  T1,x=a;
(%o12)  (a + 1)^3
(%i13)  "*****************************************************"$
(%i14)  T1,x=b;
(%o14)  (b + 1)^3
(%i15)  "*****************************************************"$
(%i16)  T1,x=1/y;
(%o16)  (1/y + 1)^3
(%i17)  "*****************************************************"$
(%i18)  T1,x=r+s;
(%o18)  (s + r + 1)^3
(%i19)  "*****************************************************"$
(%i20)
```

Wert des Terms für x = 0

Wert des Terms für x = 1

es können auch andere Terme für die Wertbelegung verwendet werden

```
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```

(%i1)
```
"*****************************************************"$
```

(%i2)
```
" Verkettung von Funktionen "$
```

(%i3)
```
"*****************************************************"$
```

(%i4)
```
u(x):=x+1;
```

(%o4) $u(x) := x + 1$

(%i5)
```
f(x):=x^2;
```

(%o5) $f(x) := x^2$

(%i6)
```
f(u(x));
```
hier werden zwei Funktionen "verkettet"

(%o6) $(x + 1)^2$

(%i7)
```
"*****************************************************"$
```

(%i8)
```
u(x):=sin(x);
```

(%o8) $u(x) := \sin(x)$

(%i9)
```
f(x):=exp(x);
```

(%o9) $f(x) := \mathrm{EXP}(x)$

(%i10)
```
f(u(x));
```

(%o10) $\%e^{\sin(x)}$

(%i11)
```
"*****************************************************"$
```

(%i12)
```
u(x):=sqrt(x);
```

(%o12) $u(x) := \sqrt{x}$

(%i13)
```
f(x):=x^2;
```

(%o13) $f(x) := x^2$

(%i14)
```
f(u(x));
```

(%o14) x

(%i15)
```
"*****************************************************"$
```

(%i16)
```
u(x):=sin(x)+cos(x);
```

(%o16) $u(x) := \sin(x) + \cos(x)$

(%i17)
```
f(x):=sqrt(x);
```

(%o17) $f(x) := \sqrt{x}$

(%i18)
```
f(u(x));
```

(%o18) $\sqrt{\sin(x) + \cos(x)}$

(%i19)
```
"*****************************************************"$
```

```
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(%i1)  "*****************************************************"$
(%i2)  " Wertetabelle einer mehrdimensionalen Funktion "$
(%i3)  "*****************************************************"$
(%i4)  f(x,y):=sin(x)*cos(y);
(%o4)   f(x , y) := sin(x) cos(y)
(%i5)  "*****************************************************"$
(%i6)  for i:1 thru 3 do for j:1 thru 3 do display(f(i,j)); f(1 , 1) =
cos(1) sin(1) f(1 , 2) = sin(1) cos(2) f(1 , 3) = sin(1) cos(3) f(2 , 1) = cos(1)
sin(2) f(2 , 2) = cos(2) sin(2) f(2 , 3) = sin(2) cos(3) f(3 , 1) = cos(1) sin(3)
f(3 , 2) = cos(2) sin(3) f(3 , 3) = cos(3) sin(3)
(%o6)   DONE
(%i7)
```

hier wird eine FOR-Schleife verwendet. Man nennt eine solche Schleife auch "Zählschleife". Eine Schleife wird mehrmals durchlaufen.

```
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(%i1)  "*****************************************************"$
(%i2)  " Berechnung einer Wertetabelle "$
(%i3)  "*****************************************************"$
(%i4)  f(x):=x^2-8*x+15;
(%o4)   f(x) := x^2 - 8 x + 15
(%i5)  "*****************************************************"$
(%i6)  " Jetzt verwenden wir eine Schleifenanweisung: "$
(%i7)  for i:-3 thru 6 do display(f(i)); f(- 3) = 48 f(- 2) = 35 f(- 1) = 24
f(0) = 15 f(1) = 8 f(2) = 3 f(3) = 0 f(4) = - 1 f(5) = 0 f(6) = 3
(%o7)   DONE
(%i8)  " Alternative ist die Verwendung von MAKELIST "$
(%i9)  makelist(f(x),x,-3,6);
(%o9)   [ 48 , 35 , 24 , 15 , 8 , 3 , 0 , - 1 , 0 , 3 ]
(%i10)
```

Wertetabellen sind für das Verständnis und die grafische Darstellung von Funktionen sehr wichtig.

```
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```

```
(%i1)  "*****************************************************"$
(%i2)  " Wertetabellen mit MAKELIST "$
(%i4)  "*****************************************************"$
(%i5)  f(n):=1/n;
```

Funktionen mit den ersten n natürlichen Zahlen als Definitionsmenge werden Folgen genannt.

$$(\%o5)\quad f(n) := \frac{1}{n}$$

```
(%i6)  makelist(f(n),n,1,10);
```

"makelist" ist sehr gut geeignet, um Folgen zu erzeugen

$$(\%o6)\quad [1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \frac{1}{8}, \frac{1}{9}, \frac{1}{10}]$$

```
(%i7)  "*****************************************************"$
(%i8)  f(i):=i;
```

$$(\%o8)\quad f(i) := i$$

```
(%i9)  makelist(f(i),i,1,10);
```

$$(\%o9)\quad [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]$$

```
(%i10)  "*****************************************************"$
(%i11)  f(n):=(n+1)/(n+2);
```

$$(\%o11)\quad f(n) := \frac{n+1}{n+2}$$

```
(%i12)  makelist(f(n),n,1,10);
```

$$(\%o12)\quad [\frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}, \frac{7}{8}, \frac{8}{9}, \frac{9}{10}, \frac{10}{11}, \frac{11}{12}]$$

```
(%i13)  "*****************************************************"$
(%i14)  f(n):=n!;
```

$$(\%o14)\quad f(n) := n!$$

```
(%i15)  makelist(f(n),n,0,10);
(%o15)  [1, 1, 2, 6, 24, 120, 720, 5040, 40320, 362880, 3628800]
(%i16)  "*****************************************************"$
(%i18)  f(i):=binom(3,i);
```

statt "binom" könnte man auch "binomial" schreiben

$$(\%o18)\quad f(i) := \binom{3}{i}$$

```
(%i19)  makelist(f(i),i,0,3);
(%o19)  [1, 3, 3, 1]
(%i20)  "*****************************************************"$
(%i21)  K(x):=x^2+8*x+36;
```

(%o21) $K(x) := x^2 + 8\,x + 36$ eine quadratische Kostenfunktion

(%i22)
```
DK(x):=K(x)/x;
```

(%o22) $DK(x) := \frac{K(x)}{x}$ Durchschnittskosten oder Stückkosten

(%i23)
```
makelist(DK(x),x,1,10);
```

(%o23) $[45, 28, 23, 21, \frac{101}{5}, 20, \frac{141}{7}, \frac{41}{2}, 21, \frac{108}{5}]$

(%i24)
```
"**************************************************"$
```

(%i25) Das ist die Tabelle der Durchschnittskosten für den Bereich x=1 bis x=10

```
(%i1)  "*************************************************"$
(%i2)  " Die Katheten eines rechtwinkeligen Dreiecks     "$
(%i3)  " sind 12 m und 16 m                              "$
(%i4)  " Berechnen Sie Fläche und Höhe                   "$
(%i5)  "*************************************************"$
(%i6)  a:12;
(%o6)   12
(%i7)  b:16;
(%o7)   16
(%i8)  "*************************************************"$
(%i9)  A:a*b/2;
(%o9)   96
(%i10)  "*************************************************"$
(%i11)  " Die Fläche ist 96 m²                            "$
(%i12)  "*************************************************"$
(%i13)  c:sqrt(a^2+b^2);
(%o13)   20
(%i14)  g:A=c*h/2;
(%o14)   96 = 10 h
(%i15)  solve(g,h);
(%o15)   [ h = 48/5 ]
(%i16)  "*************************************************"$
(%i17)  " Die Höhe ist 9,6 m                              "$
(%i18)  "*************************************************"$
(%i19)
```

Gegeben sind die Katheten eines rechtwinkeligen Dreiecks

Flächenberechnung

das ist der Pythagoräische Lehrsatz

```
(%i1)  "*************************************************"$
(%i2)  " 5 Allgemeine Dreiecke nach Dreiecksungleichung  "$
(%i3)  "*************************************************"$
(%i4)  a:[4,3,6,9,8];
(%o4)   [ 4 , 3 , 6 , 9 , 8 ]
(%i5)  b:[3,4,2,5,6];
(%o5)   [ 3 , 4 , 2 , 5 , 6 ]
(%i6)  c:a+b-1;                    a + b > c (Dreiecksungleichung)
(%o6)   [ 6 , 6 , 7 , 13 , 13 ]
(%i7)  "*************************************************"$
(%i8)  " Dreiecksungleichung: a+b < c                    "$
(%i9)  "*************************************************"$
(%i10)  u[1]:a[1]+b[1]+c[1];   indizierte Variable: direkter Zugriff auf einzelne Listenelemente
(%o10)   13
(%i11)  makelist(u[i]:a[i]+b[i]+c[i],i,1,5);
(%o11)   [ 13 , 13 , 15 , 27 , 27 ]
(%i12)  "*************************************************"$
(%i13)  " Liste des jeweiligen Umfangs                   "$
(%i16)  "*************************************************"$
(%i17)  u[1];
(%o17)   13
(%i18)  u[2];
(%o18)   13
(%i19)  u[3];
(%o19)   15
(%i20)  makelist(s[i]:u[i]/2,i,1,5);
(%o20)   [13/2, 13/2, 15/2, 27/2, 27/2]   braucht man für die Heronsche Formel
(%i21)  "*************************************************"$
(%i22)  " s ist der halbe Umfang                         "$
(%i23)  "*************************************************"$
(%i24)  Berechnung der Flächen nach der Heronschen Formel
makelist(A[i]:sqrt(s[i]*(s[i]-a[i])*(s[i]-b[i])*(s[i]-c[i])),i,1,5);
(%o24)   [sqrt(455)/4, sqrt(455)/4, 3*sqrt(55)/4, 9*sqrt(51)/4, 9*sqrt(55)/4]
(%i26)  "*************************************************"$
(%i27)  " Das sollten die jeweiligen Flächen sein         "$
(%i28)  "*************************************************"$
(%i29)
```

```
(%i1)  "*****************************************************"$
(%i2)  " 5 Allgemeine Dreiecke nach Dreiecksungleichung     "$
(%i3)  "*****************************************************"$
(%i4)  a:[4,3,6,9,8];
(%o4)   [ 4 , 3 , 6 , 9 , 8 ]
(%i5)  b:[3,4,2,5,6];
(%o5)   [ 3 , 4 , 2 , 5 , 6 ]
(%i6)  c:a+b-1;
(%o6)   [ 6 , 6 , 7 , 13 , 13 ]
(%i7)  "*****************************************************"$
(%i8)  " Dreiecksungleichung: a+b < c                       "$
(%i9)  "*****************************************************"$
(%i10)  u[1]:a[1]+b[1]+c[1];
(%o10)   13
(%i11)  makelist(u[i]:a[i]+b[i]+c[i],i,1,5);
(%o11)   [ 13 , 13 , 15 , 27 , 27 ]
(%i12)  "*****************************************************"$
(%i13)  " Liste des jeweiligen Umfangs                      "$
(%i16)  "*****************************************************"$
(%i17)  u[1];
(%o17)   13
(%i18)  u[2];
(%o18)   13
(%i19)  u[3];
(%o19)   15
(%i20)  makelist(s[i]:u[i]/2,i,1,5);
(%o20)   [13/2, 13/2, 15/2, 27/2, 27/2]
(%i21)  "*****************************************************"$
(%i22)  " s ist der halbe Umfang                            "$
(%i23)  "*****************************************************"$
(%i24)
makelist(A[i]:sqrt(s[i]*(s[i]-a[i])*(s[i]-b[i])*(s[i]-c[i])),i,1,5);
(%o24)   [sqrt(455)/4, sqrt(455)/4, 3*sqrt(55)/4, 9*sqrt(51)/4, 9*sqrt(55)/4]
(%i26)  "*****************************************************"$
(%i27)  " Das sollten die jeweiligen Flächen sein            "$
(%i28)  "*****************************************************"$
(%i29)  u1:a+b+c;
```

```
(%o29)   [ 13 , 13 , 15 , 27 , 27 ]
(%i30)  s1:u1/2;
```

$$(\%o30)\quad \left[\frac{13}{2},\frac{13}{2},\frac{15}{2},\frac{27}{2},\frac{27}{2}\right]$$

```
(%i32)  A:sqrt(s1*(s1-a)*(s1-b)*(s1-c));
```

$$(\%o32)\quad \left[\frac{\sqrt{455}}{4},\frac{\sqrt{455}}{4},\frac{3\sqrt{55}}{4},\frac{9\sqrt{51}}{4},\frac{9\sqrt{55}}{4}\right]$$

```
(%i33)  "*****************************************************"$
(%i34)  " Mit Listenverarbeitung kommt man sehr elegant      "$
(%i35)  " zu den gleichen Resultaten                         "$
(%i37)  "*****************************************************"$
(%i38)
```

Listenverarbeitung ist ein zentrales Thema in
1.) der Programmiersprache LISP
2.) im CAS Maxima

```
(%i1)  "*****************************************************"$
(%i2)  " Seiten eines gleichschenkeligen Dreiecks bekannt    "$
(%i3)  "*****************************************************"$
(%i4)  a:6;
(%o4)   6
(%i5)  c:4;
(%o5)   4
(%i6)  "*****************************************************"$
(%i7)  hc:sqrt(a^2-(c/2)^2);
(%o7)   4 sqrt(2)
(%i8)  "*****************************************************"$
(%i9)  " Höhe auf c                                          "$
(%i10)  "*****************************************************"$
(%i11)  A:c*hc/2;
(%o11)   8 sqrt(2)
(%i12)  "*****************************************************"$
(%i13)  ha:2*A/a;
                          8 sqrt(2)
(%o13)                    ---------
                              3
(%i14)  "*****************************************************"$
(%i15)  U:2*a+c;
(%o15)   16
(%i16)  "*****************************************************"$
(%i17)  "*****************************************************"$
(%i18)  a:makelist(5*k+1,k,1,5);
(%o18)   [ 6 , 11 , 16 , 21 , 26 ]
(%i19)  c:makelist(6*k-2,k,1,5);
(%o19)   [ 4 , 10 , 16 , 22 , 28 ]
(%i20)  "*****************************************************"$
(%i21)  hc:sqrt(a^2-(c/2)^2);
(%o21)   [ 4 sqrt(2) , 4 sqrt(6) , 8 sqrt(3) , 8 sqrt(5) , 4 sqrt(30) ]
(%i22)  A:c*hc/2;
(%o22)   [ 8 sqrt(2) , 20 sqrt(6) , 64 sqrt(3) , 88 sqrt(5) , 56 sqrt(30) ]
(%i23)  ha:2*A/a;
                 8 sqrt(2)  40 sqrt(6)            176 sqrt(5)  56 sqrt(30)
(%o23)   [       ---------, ---------, 8 sqrt(3), ---------, ---------  ]
                     3          11                    21           13
(%i24)  U:2*a+c;
(%o24)   [ 16 , 32 , 48 , 64 , 80 ]
```

```
(%i1)  "*****************************************************"$
(%i2)  " Gleichschenkeliges Dreieck: Schenkel und Höhe      "$
(%i4)  "*****************************************************"$
(%i5)  a:32.5;
(%o5)   32.5
(%i6)  h:31.5;
(%o6)   31.5
(%i7)  "*****************************************************"$
(%i8)  g:a^2=h^2+(c/2)^2;
                                    2
(%o8)   1056.25 = c/4 + 992.25
(%i9)  solve(g,c);
RAT replaced 64.0 by 64//1 = 64.0
(%o9)   [ c = - 16 , c = 16 ]
(%i10)  c:16;
(%o10)   16
(%i11)  "*****************************************************"$
(%i12)  A:c*h/2;
(%o12)   252.0
(%i13)  "*****************************************************"$
(%i14)  U:2*a+c;
(%o14)   81.0
(%i15)  "*****************************************************"$
(%i16)  ha:2*A/a;
(%o16)   15.507692307692308
(%i18)
```

```
(%i1)  "*****************************************************"$
(%i2)  " Gleichschenkelige Dreiecke                          "$
(%i3)  "*****************************************************"$
(%i4)  c:[5,4,6,8,14];
(%o4)   [ 5 , 4 , 6 , 8 , 14 ]
(%i5)  h:[3,4,5,6,4];
(%o5)   [ 3 , 4 , 5 , 6 , 4 ]
(%i6)  "*****************************************************"$
(%i7)  " Man kennt also die Grundlinie und die Höhe          "$
(%i8)  "*****************************************************"$
(%i9)  a:sqrt(h^2+(c/2)^2);
(%o9)   [sqrt(61)/2 , 2 sqrt(5) , sqrt(34) , 2 sqrt(13) , sqrt(65) ]
(%i10)  "*****************************************************"$
(%i11)  " Das sind die Schenkel                               "$
(%i12)  "*****************************************************"$
(%i13)  U:2*a+c;
(%o13)   [sqrt(61) + 5 , 4 sqrt(5) + 4 , 2 sqrt(34) + 6 , 4 sqrt(13) + 8 , 2 sqrt(65) + 14 ]
(%i14)  "*****************************************************"$
(%i15)  " Das ist der jeweilige Umfang                        "$
(%i16)  "*****************************************************"$
(%i20)
```

```
(%i1)  "*****************************************************"$
(%i2)  " Gleichschenkelige Dreiecke                          "$
(%i3)  "*****************************************************"$
(%i4)  c:makelist(7*k-1,k,1,5);
(%o4)   [ 6 , 13 , 20 , 27 , 34 ]
(%i5)  hc:makelist(6*k-2,k,1,5);
(%o5)   [ 4 , 10 , 16 , 22 , 28 ]
(%i6)  "*****************************************************"$
(%i7)  a:sqrt(hc^2+(c/2)^2);
```

$$(\%o7)\quad \left[5\ ,\ \frac{\sqrt{569}}{2}\ ,\ 2\sqrt{89}\ ,\ \frac{\sqrt{2665}}{2}\ ,\ \sqrt{1073}\right]$$

```
(%i8)  "*****************************************************"$
(%i9)  A:c*hc/2;
(%o9)   [ 12 , 65 , 160 , 297 , 476 ]
(%i10)  "*****************************************************"$
(%i11)  U:2*a+c;
```

$$(\%o11)\quad \left[16\ ,\ \sqrt{569}+13\ ,\ 4\sqrt{89}+20\ ,\ \sqrt{2665}+27\ ,\ 2\sqrt{1073}+34\right]$$

```
(%i12)  "*****************************************************"$
(%i13)  ha:2*A/a;
```

$$(\%o13)\quad \left[\frac{24}{5}\ ,\ \frac{260}{\sqrt{569}}\ ,\ \frac{160}{\sqrt{89}}\ ,\ \frac{1188}{\sqrt{2665}}\ ,\ \frac{952}{\sqrt{1073}}\right]$$

```
(%i14)  "*****************************************************"$
(%i15)
```

```
(%i1)  "*****************************************************"$
(%i2)  " Gleichseitiges Dreieck - Heronsche Formel           "$
(%i3)  "*****************************************************"$
(%i4)  a:x;
(%o4)   x
(%i5)  b:x;
(%o5)   x
(%i6)  c:x;
(%o6)   x
(%i7)  "*****************************************************"$
(%i8)  U:a+b+c;
(%o8)   3 x
(%i9)  s:U/2;
                3 x
(%o9)           ---
                 2
(%i10)  "*****************************************************"$
(%i11)  A:sqrt(s*(s-a)*(s-b)*(s-c));
                sqrt(3) x^2
(%o11)          -----------
                     4
(%i12)  "*****************************************************"$
(%i15)  " Flächenformel für das gleichseitige Dreieck        "$
(%i16)  "*****************************************************"$
(%i17)
```

```
(%i1)  "*****************************************************"$
(%i2)  " In einem rechtwinkeligen Dreieck ist der Flächen-  "$
(%i3)  " inhalt 120 cm² und die Seitenlänge a 24 cm.        "$
(%i4)  " Wie groß sind c und b                              "$
(%i5)  "*****************************************************"$
(%i6)  A:120;
(%o6)   120
(%i8)  a:24;
(%o8)   24
(%i10)  "*****************************************************"$
(%i11)  g:A=a*b/2;
(%o11)   120 = 12 b
(%i12)  solve(g,b);
(%o12)   [ b = 10 ]
(%i13)  b:10;
(%o13)   10
(%i14)  "*****************************************************"$
(%i15)  c:sqrt(a^2+b^2);
(%o15)   26
(%i16)  "*****************************************************"$
(%i17)
```

```
(%i1)  "*****************************************************"$
(%i2)  " Lehrsatz Pythagoras - Listenverarbeitung           "$
(%i3)  "*****************************************************"$
(%i4)  a:[3,6,7,9,13];
(%o4)   [ 3 , 6 , 7 , 9 , 13 ]
(%i5)  b:[4,8,12,10,14];
(%o5)   [ 4 , 8 , 12 , 10 , 14 ]
(%i6)  "*****************************************************"$
(%i7)  " Listen der Katheten                                "$
(%i8)  "*****************************************************"$
(%i9)  c:sqrt(a^2+b^2);
(%o9)   [ 5 , 10 , sqrt(193) , sqrt(181) , sqrt(365) ]
(%i10)  "*****************************************************"$
(%i11)  " Das sind die Hypothenusen                          "$
(%i12)  "*****************************************************"$
(%i15)  "Aus a*b/2=c*h/2 folgt h=a*b/c                       "$
(%i16)  "*****************************************************"$
(%i17)  h:a*b/c;
                                12   24   84    90    182
(%o17)                         [--, --, ----, ----, ----]
                                5    5   sqrt  sqrt  sqrt
                                         (193) (181) (365)
(%i18)  "*****************************************************"$
(%i19)  " Das ist die Liste der Höhen                        "$
(%i20)  "*****************************************************"$
(%i21)
```

```
(%i1)  "*************************************************"$
(%i2)  " Rechtwinkelige Dreiecke - Listenverarbeitung      "$
(%i3)  "*************************************************"$
(%i4)  a:[3,6,9,12,15];
(%o4)   [ 3 , 6 , 9 , 12 , 15 ]
(%i5)  b:[4,8,12,16,20];
(%o5)   [ 4 , 8 , 12 , 16 , 20 ]
(%i6)  "*************************************************"$
(%i7)  " Liste der Katheten                               "$
(%i8)  "*************************************************"$
(%i9)  c:sqrt(a^2+b^2);
(%o9)   [ 5 , 10 , 15 , 20 , 25 ]
(%i10)  "*************************************************"$
(%i11)  h:a*b/c;
(%o11)   [ 12/5 , 24/5 , 36/5 , 48/5 , 12 ]
(%i12)  "*************************************************"$
(%i13)  p:a^2/c;
(%o13)   [ 9/5 , 18/5 , 27/5 , 36/5 , 9 ]
(%i14)  "*************************************************"$
(%i15)  q:b^2/c;
(%o15)   [ 16/5 , 32/5 , 48/5 , 64/5 , 16 ]
(%i16)  "*************************************************"$
(%i17)  c;
(%o17)   [ 5 , 10 , 15 , 20 , 25 ]
(%i18)  c1:p+q;
(%o18)   [ 5 , 10 , 15 , 20 , 25 ]
(%i19)  "*************************************************"$
(%i20)  " Probe stimmt                                    "$
(%i21)  "*************************************************"$
(%i22)  A:a*b/2;
(%o22)   [ 6 , 24 , 54 , 96 , 150 ]
(%i23)  A1:c*h/2;
(%o23)   [ 6 , 24 , 54 , 96 , 150 ]
(%i24)  "*************************************************"$
(%i25)  U:a+b+c;
(%o25)   [ 12 , 24 , 36 , 48 , 60 ]
```

```
(%i26)  s:U/2;
(%o26)   [ 6 , 12 , 18 , 24 , 30 ]
(%i27)  A3:sqrt(s*(s-a)*(s-b)*(s-c));
(%o27)   [ 6 , 24 , 54 , 96 , 150 ]
(%i28)  "*****************************************************"$
(%i29)  " Probe nach HERON stimmt                             "$
(%i30)  "*****************************************************"$
(%i31)
```

```
(%i1)  "*****************************************************"$
(%i2)  " Rechtwinkelige Dreiecke - Listenverarbeitung       "$
(%i3)  "*****************************************************"$
(%i4)  a:[3,6,9,12,15];
(%o4)   [ 3 , 6 , 9 , 12 , 15 ]
(%i8)  h:[12/5,24/5,36/5,48/5,12];
(%o8)   [ 12/5 , 24/5 , 36/5 , 48/5 , 12 ]
(%i9)  "*****************************************************"$
(%i10)  p:sqrt(a^2-h^2);
(%o10)   [ 9/5 , 18/5 , 27/5 , 36/5 , 9 ]
(%i11)  "*****************************************************"$
(%i12)  c:a^2/p;
(%o12)   [ 5 , 10 , 15 , 20 , 25 ]
(%i13)  "*****************************************************"$
(%i14)  q:c-p;
(%o14)   [ 16/5 , 32/5 , 48/5 , 64/5 , 16 ]
(%i15)  "*****************************************************"$
(%i16)  b:sqrt(c*q);
(%o16)   [ 4 , 8 , 12 , 16 , 20 ]
(%i17)  b1:sqrt(c^2-a^2);
(%o17)   [ 4 , 8 , 12 , 16 , 20 ]
(%i18)  "*****************************************************"$
(%i19)  A:a*b/2;
(%o19)   [ 6 , 24 , 54 , 96 , 150 ]
(%i20)  A1:c*h/2;
(%o20)   [ 6 , 24 , 54 , 96 , 150 ]
(%i21)  "*****************************************************"$
(%i22)  "*****************************************************"$
(%i23)
```

```
(%i1)  "*****************************************************"$
(%i2)  " Listenverarbeitung rechtwinkeliges Dreieck          "$
(%i3)  "*****************************************************"$
(%i4)  b:makelist(99*i,i,1,5);
(%o4)   [ 99 , 198 , 297 , 396 , 495 ]
(%i5)  b;
(%o5)   [ 99 , 198 , 297 , 396 , 495 ]
(%i6)  A:makelist(990*i^2,i,1,5);
(%o6)   [ 990 , 3960 , 8910 , 15840 , 24750 ]
(%i7)  A;
(%o7)   [ 990 , 3960 , 8910 , 15840 , 24750 ]
(%i8)  "*****************************************************"$
(%i9)  a:2*A/b;
(%o9)   [ 20 , 40 , 60 , 80 , 100 ]
(%i10)  "*****************************************************"$
(%i11)  c:sqrt(a^2+b^2);
(%o11)   [ 101 , 202 , 303 , 404 , 505 ]
(%i12)  "*****************************************************"$
(%i13)  h:a*b/c;
(%o13)   [1980/101 , 3960/101 , 5940/101 , 7920/101 , 9900/101]
(%i14)  "*****************************************************"$
(%i15)  p:a^2/c;
(%o15)   [400/101 , 800/101 , 1200/101 , 1600/101 , 2000/101]
(%i16)  q:b^2/c;
(%o16)   [9801/101 , 19602/101 , 29403/101 , 39204/101 , 49005/101]
(%i17)  c1:p+q;
(%o17)   [ 101 , 202 , 303 , 404 , 505 ]
(%i18)  "*****************************************************"$
(%i19)  " Merke: wenn sich eine Kathete verdoppelt,          "$
(%i20)  " vervierfacht sich die Fläche                       "$
(%i21)  "*****************************************************"$
(%i22)
```

```
(%i1)  "*************************************************"$
(%i2)  " Rechtwinkeliges Dreieck - Listenverarbeitung      "$
(%i3)  "*************************************************"$
(%i4)  a:makelist(18*i,i,1,5);
(%o4)   [ 18 , 36 , 54 , 72 , 90 ]
(%i5)  A:makelist(720*i^2,i,1,5);
(%o5)   [ 720 , 2880 , 6480 , 11520 , 18000 ]
(%i6)  "*************************************************"$
(%i7)  b:2*A/a;
(%o7)   [ 80 , 160 , 240 , 320 , 400 ]
(%i8)  "*************************************************"$
(%i9)  c:sqrt(a^2+b^2);
(%o9)   [ 82 , 164 , 246 , 328 , 410 ]
(%i10)  "*************************************************"$
(%i11)  h:a*b/c;
(%o11)   [720/41 , 1440/41 , 2160/41 , 2880/41 , 3600/41]
(%i12)  "*************************************************"$
(%i13)  p:a^2/c;
(%o13)   [162/41 , 324/41 , 486/41 , 648/41 , 810/41]
(%i14)  q:b^2/c;
(%o14)   [3200/41 , 6400/41 , 9600/41 , 12800/41 , 16000/41]
(%i15)  c1:p+q;
(%o15)   [ 82 , 164 , 246 , 328 , 410 ]
(%i16)
```

```
(%i1) "*****************************************************"$
(%i2) " Gegeben sind Hypothenuse c und Kathete a            "$
(%i3) " Berechne den Umfang mittels Listenverarbeitung      "$
(%i4) "*****************************************************"$
(%i5) c:[10,13,17,33, 25];
(%o5) [ 10 , 13 , 17 , 33 , 25 ]
(%i6) a:c-2;
(%o6) [ 8 , 11 , 15 , 31 , 23 ]
(%i7) "*****************************************************"$
(%i8) b:sqrt(c^2-a^2);
```

(%o8) $[\,6\,,\,4\sqrt{3}\,,\,8\,,\,8\sqrt{2}\,,\,4\sqrt{6}\,]$

```
(%i9) "*****************************************************"$
(%i10) u:a+b+c;
```

(%o10) $[\,24\,,\,4\sqrt{3}+24\,,\,40\,,\,8\sqrt{2}+64\,,\,4\sqrt{6}+48\,]$

```
(%i11) "*****************************************************"$
(%i12) " Wie lauten die Flächen?                             "$
(%i13) "*****************************************************"$
(%i14) A:a*b/2;
```

(%o14) $[\,24\,,\,22\sqrt{3}\,,\,60\,,\,124\sqrt{2}\,,\,46\sqrt{6}\,]$

```
(%i15) "*****************************************************"$
(%i16) " Heronsche Formel                                    "$
(%i17) "*****************************************************"$
(%i18) s:u/2;
```

(%o18) $\left[\,12\,,\,\frac{4\sqrt{3}+24}{2}\,,\,20\,,\,\frac{8\sqrt{2}+64}{2}\,,\,\frac{4\sqrt{6}+48}{2}\,\right]$

```
(%i19) F:sqrt(s*(s-a)*(s-b)*(s-c));
```

(%o19) $\left[\,24\,,\,\frac{\sqrt{4\sqrt{3}+24}\sqrt{\frac{4\sqrt{3}+24}{2}-13}\sqrt{\frac{4\sqrt{3}+24}{2}-11}\sqrt{\frac{4\sqrt{3}+24}{2}-4\sqrt{3}}}{\sqrt{2}}\,,\,60\,,\right.$

$\frac{\sqrt{8\sqrt{2}+64}\sqrt{\frac{8\sqrt{2}+64}{2}-33}\sqrt{\frac{8\sqrt{2}+64}{2}-31}\sqrt{\frac{8\sqrt{2}+64}{2}-8\sqrt{2}}}{\sqrt{2}}\,,$

$\left.\frac{\sqrt{4\sqrt{6}+48}\sqrt{\frac{4\sqrt{6}+48}{2}-25}\sqrt{\frac{4\sqrt{6}+48}{2}-23}\sqrt{\frac{4\sqrt{6}+48}{2}-4\sqrt{6}}}{\sqrt{2}}\,\right]$

```
(%i22) unterschied:A-F,numer;
```

(%o22) $[\,0\,,\,1.4210854715202\;10^{-14}\,,\,0\,,\,-\,1.1368683772161\;10^{-13}\,,\,-\,4.263256414560601\;10^{-14}\,]$

```
(%i23) "*****************************************************"$
```

```
(%i24)  " Alle Unterschiede sind NULL                                    "$
(%i25)  "*************************************************************"$
(%i26)
```

```
(%i1)  "*****************************************************"$
(%i2)  " Winkel im recthwinkeligen Dreieck                    "$
(%i3)  "*****************************************************"$
(%i4)  " Beispiel 1:                                          "$
(%i5)  " der Winkel Alpha ist doppelt so groß                 "$
(%i6)  " wie ger Winkel Beta                                  "$
(%i8)  " Beispiel 2:"$
(%i13)  " der Winkel Alpha ist dreimal so groß                 "$
(%i14)  " wie der Winkel Beta                                  "$
(%i15)  "*****************************************************"$
(%i16)  g1:gamma=90;
(%o16)   gamma = 90
(%i18)  g2:alpha+beta+gamma=180;
(%o18)   gamma + BETA + ALPHA = 180
(%i20)  g3:alpha=2*beta;
(%o20)   ALPHA = 2 BETA
(%i21)  "*****************************************************"$
(%i22)  solve([g1,g2,g3],[alpha,beta,gamma]);
(%o22)   [ [ ALPHA = 60 , BETA = 30 , gamma = 90 ] ]
(%i23)  "*****************************************************"$
(%i24)  " Ergebnis von Beispiel 1                              "$
(%i25)  "*****************************************************"$
(%i26)  g3:alpha=3*beta;
(%o26)   ALPHA = 3 BETA
(%i27)  solve([g1,g2,g3],[alpha,beta,gamma]);
                                  135          45
(%o27)   [ [ ALPHA = ---, BETA = --, gamma = 90 ] ]
                                   2           2
(%i28)  "*****************************************************"$
(%i29)  " Ergebnis von Beispiel 2                              "$
(%i30)  "*****************************************************"$
(%i31)
```

```
(%i1)  "*****************************************************"$
(%i2)  " Rechtwinkeliges Dreieck, Listenverarbeitung         "$
(%i3)  "*****************************************************"$
(%i4)  a:makelist(5*i,i,1,5);
(%o4)   [ 5 , 10 , 15 , 20 , 25 ]
(%i5)  c:makelist(13*i,i,1,5);
(%o5)   [ 13 , 26 , 39 , 52 , 65 ]
(%i6)  "*****************************************************"$
(%i7)  b:sqrt(c^2-a^2);
(%o7)   [ 12 , 24 , 36 , 48 , 60 ]
(%i8)  "*****************************************************"$
(%i9)  h:a*b/c;
(%o9)   [60/13 , 120/13 , 180/13 , 240/13 , 300/13]
(%i10)  "*****************************************************"$
(%i11)  p:a^2/c;
(%o11)   [25/13 , 50/13 , 75/13 , 100/13 , 125/13]
(%i12)  q:b^2/c;
(%o12)   [144/13 , 288/13 , 432/13 , 576/13 , 720/13]
(%i13)  c1:p+q;
(%o13)   [ 13 , 26 , 39 , 52 , 65 ]
(%i14)  "*****************************************************"$
(%i15)  " Summenprobe passt                                  "$
(%i16)  "*****************************************************"$
(%i17)  A:a*b/2;
(%o17)   [ 30 , 120 , 270 , 480 , 750 ]
(%i18)  A1:c*h/2;
(%o18)   [ 30 , 120 , 270 , 480 , 750 ]
(%i19)  "*****************************************************"$
(%i20)  "*****************************************************"$
(%i21)
```

```
(%i1)  "*****************************************************"$
(%i2)  " Höhen und Flächen von gleichseitigen Dreiecken     "$
(%i3)  "*****************************************************"$
(%i4)  a:[10,20,30,40,50];
(%o4)   [ 10 , 20 , 30 , 40 , 50 ]
(%i5)  "*****************************************************"$
(%i6)  h:a/2*sqrt(3);
(%o6)   [ 5 sqrt(3) , 10 sqrt(3) , 15 sqrt(3) , 20 sqrt(3) , 25 sqrt(3) ]
(%i7)  "*****************************************************"$
(%i8)  " Das sind die Höhen                                 "$
(%i9)  "*****************************************************"$
(%i10)  A:a^2/4*sqrt(3);
(%o10)   [ 25 sqrt(3) , 100 sqrt(3) , 225 sqrt(3) , 400 sqrt(3) , 625 sqrt(3) ]
(%i11)  "*****************************************************"$
(%i12)  " Das sind die Flächen                               "$
(%i13)  "*****************************************************"$
(%i14)
```

```
(%i1)  "*****************************************************"$
(%i2)  " Eine konvergente Reihe                               "$
(%i3)  "*****************************************************"$
(%i4)  f(x):=1/2^x;
```

$$(\%o4)\quad f(x) := \frac{1}{2^x}$$

```
(%i7)  "*****************************************************"$
(%i8)  ps(n):=sum(f(x),x,1,n);
```

$$(\%o8)\quad PS(n) := SUM(f(x),\ x\ ,\ 1\ ,\ n)$$

```
(%i9)  "*****************************************************"$
(%i10)  ps(10);
```

$$(\%o10)\quad \frac{1023}{1024}$$

```
(%i11)  %,numer;
(%o11)   0.9990234375
(%i12)  "*****************************************************"$
(%i13)  ps(100),numer;
(%o13)   1.0
(%i14)  "*****************************************************"$
(%i16)  ps(1000),numer;
(%o16)   1.0
(%i17)  "*****************************************************"$
(%i18)  limit(ps(n),n,INF);
```

$$(\%o18)\quad \lim_{n \to \inf} \sum_{x=1}^{n} f(x)$$

```
(%i19)  sum(1/2^x,x,1,INF);
```

$$(\%o19)\quad \sum_{x=1}^{\inf} \frac{1}{2^x}$$

```
(%i20)  %,simpsum;
(%o20)   1
(%i21)  "*****************************************************"$
(%i22)  " Achtung: hier gibt es Schwierigkeiten bei der       "$
(%i23)  " Berechnung                                          "$
(%i24)  "*****************************************************"$
(%i25)
```

```
(%i1)  f(x):=(-1)^x*1/2^x;
```

$$(\%o1)\quad f(x) := \frac{(-1)^x\ 1}{2^x}$$

```
(%i2)  makelist(f(x),x,1,10);
```

$$(\%o2)\quad \left[-\frac{1}{2},\frac{1}{4},-\frac{1}{8},\frac{1}{16},-\frac{1}{32},\frac{1}{64},-\frac{1}{128},\frac{1}{256},-\frac{1}{512},\frac{1}{1024}\right]$$

```
(%i3)  sum(f(x),x,1,10);
```

$$(\%o3)\quad -\frac{341}{1024}$$

```
(%i4)  "**************************************************"$
(%i5)  sum(f(x),x,1,50);
```

$$(\%o5)\quad -\frac{375299968947541}{1125899906842624}$$

```
(%i6)  %,numer;
(%o6)   - 0.33333333333333304
(%i7)  "**************************************************"$
(%i9)  sum(f(x),x,1,1000),numer;
(%o9)   - 0.33333333333333337
(%i10)  sum(f(x),x,1,10000),numer;
(%o10)   - 0.33333333333333337
(%i11)  "**************************************************"$
(%i12)  " Die Reihe ist konvergent                          "$
(%i13)  "**************************************************"$
(%i14)   Der Grenzwert der konvergenten Reihe ist 1/3
```

```
(%i1)  "*****************************************************"$
(%i2)  " Partialsummen einer divergenten Reihe                "$
(%i3)  "*****************************************************"$
(%i4)  f(x):=1/x;
```

$$(\%o4)\quad f(x) := \frac{1}{x}$$

```
(%i5)  "*****************************************************"$
(%i6)  sum(f(x),x,1,10);
```

$$(\%o6)\quad \frac{7381}{2520}$$

```
(%i7)  %,numer;
(%o7)   2.9289682539682538
(%i8)  "*****************************************************"$
(%i9)  sum(f(x),x,1,100),numer;
(%o9)   5.1873775176396206
(%i10)  "*****************************************************"$
(%i15)  sum(f(x),x,1,200),numer;
(%o15)   5.8780309481214461
(%i16)  "*****************************************************"$
(%i18)  sum(f(x),x,1,300),numer;
(%o18)   6.2826638802995021
(%i19)  "*****************************************************"$
(%i20)  sum(f(x),x,1,1000),numer;
(%o20)   7.4854708605503433
(%i21)  "*****************************************************"$
(%i23)  sum(f(x),x,1,10000),numer;
(%o23)   9.7876060360443446
(%i24)  "*****************************************************"$
(%i27)  wahrscheinlich ist die Reihe divergent
```

```
(%i1)  "*****************************************************"$
(%i2)  " Fläche und Umfang des Kreises - Listenverarbeitung "$
(%i3)  "*****************************************************"$
(%i4)  r:[10,20,30,40,50];
(%o4)   [ 10 , 20 , 30 , 40 , 50 ]
(%i5)  "*****************************************************"$
(%i6)  U:2*r*%pi;
(%o6)   [ 20 %pi , 40 %pi , 60 %pi , 80 %pi , 100 %pi ]
(%i7)  "*****************************************************"$
(%i8)  " Umfang der Kreise                                   "$
(%i9)  "*****************************************************"$
(%i10)  A:r^2*%pi;
(%o10)   [ 100 %pi , 400 %pi , 900 %pi , 1600 %pi , 2500 %pi ]
(%i11)  "*****************************************************"$
(%i12)  " Fläche der Kreise                                  "$
(%i13)  "*****************************************************"$
(%i14)  Flaechensumme:sum(A[k],k,1,5);
(%o14)   5500 %pi
(%i15)  Umfangsumme:sum(U[k],k,1,5);
(%o15)   300 %pi
(%i16)  "*****************************************************"$
(%i17)  " Summen von Fläche und Umfang                       "$
(%i18)  "*****************************************************"$
(%i19)
```

man beachte die Schreibweise der Kreiszahl

```
(%i1)  "*****************************************************"$
(%i2)  " Pythagoräischer Lehrsatz und Kathetensätze          "$
(%i4)  "*****************************************************"$
(%i5)  ks1:a^2=p*c;
```

$$a^2 = c\ p \quad \text{(\%o5)}$$

```
(%i6)  ks2:b^2=q*c;
```

$$b^2 = c\ q \quad \text{(\%o6)}$$

```
(%i7)  pls:ks1+ks2;
```

$$b^2 + a^2 = c\ q + c\ p \quad \text{(\%o7)}$$

```
(%i8)  "*****************************************************"$
(%i9)  " p+q=c                                               "$
(%i10)  "*****************************************************"$
(%i11)  pls:pls/c;
```

$$\frac{b^2 + a^2}{c} = \frac{c\ q + c\ p}{c} \quad \text{(\%o11)}$$

```
(%i12)  pls:expand(pls);
```

$$\frac{b^2}{c} + \frac{a^2}{c} = q + p \quad \text{(\%o12)}$$

```
(%i13)  "*****************************************************"$
(%i14)  " Wenn man p+q=c setzt, ist der P. Lehrsatz bewiesen "$
(%i15)  "*****************************************************"$
(%i16)
```

```
(%i1)  "*************************************************"$
(%i2)  " Flächen und Diagonalen von Quadraten            "$
(%i3)  "*************************************************"$
(%i4)  a:[10,20,30,40,50];
(%o4)   [ 10 , 20 , 30 , 40 , 50 ]
(%i5)  "*************************************************"$
(%i6)  d:a*sqrt(2);
(%o6)   [ 10 sqrt(2) , 20 sqrt(2) , 30 sqrt(2) , 40 sqrt(2) , 50 sqrt(2) ]
(%i7)  "*************************************************"$
(%i8)  " Das sind die Diagonalen                         "$
(%i9)  "*************************************************"$
(%i10)  A:a^2;
(%o10)   [ 100 , 400 , 900 , 1600 , 2500 ]
(%i11)  "*************************************************"$
(%i12)  " Das sind die Flächen                           "$
(%i13)  "*************************************************"$
(%i14)  Flaechensumme:sum(A[k],k,1,5);
(%o14)   5500
(%i15)  "*************************************************"$
(%i16)  " Das ist die Flächensumme                       "$
(%i17)  "*************************************************"$
(%i18)
```

```
(%i1)  "*************************************************"$
(%i2)  " Flächen und Diagonalen von Rechtecken           "$
(%i3)  "*************************************************"$
(%i4)  a:[10,20,30,40,50];
(%o4)   [ 10 , 20 , 30 , 40 , 50 ]
(%i5)  b:a-3;
(%o5)   [ 7 , 17 , 27 , 37 , 47 ]
(%i6)  "*************************************************"$
(%i7)  " Die Breiten sind um 3 cm kürzer                 "$
(%i8)  "*************************************************"$
(%i9)  A:a*b;
(%o9)   [ 70 , 340 , 810 , 1480 , 2350 ]
(%i10)  "*************************************************"$
(%i11)  d:sqrt(a^2+b^2);
(%o11)   [ sqrt(149) , sqrt(689) , 3 sqrt(181) , sqrt(2969) , sqrt(4709) ]
(%i12)  "*************************************************"$
(%i13)  Flaechensumme:sum(A[i],i,1,5);
(%o13)   5050
(%i14)
```

```
wxMaxima 0.6.4 http://wxmaxima.sourceforge.net
Maxima 5.9.1 http://maxima.sourceforge.net
Using Lisp Kyoto Common Lisp GCL 2.6.5 (aka GCL)
Distributed under the GNU Public License. See the file COPYING.
Dedicated to the memory of William Schelter.
This is a development version of Maxima. The function bug_report()
provides bug reporting information.
(%i1)  "*************************************************"$
(%i2)  " Größter gemeinsamer Teiler                      "$
(%i3)  "*************************************************"$
(%i4)  p1:expand((x-1)*(x+2)*(x-3)*(x+4));
(%o4)   x^4 + 2 x^3 - 13 x^2 - 14 x + 24
(%i5)  p2:p1*(x-5);
(%o5)   (x - 5) (x^4 + 2 x^3 - 13 x^2 - 14 x + 24)
(%i6)  p2:expand(p1*(x-5));
(%o6)   x^5 - 3 x^4 - 23 x^3 + 51 x^2 + 94 x - 120
(%i7)  gcd(p1,p2);
(%o7)   x^4 + 2 x^3 - 13 x^2 - 14 x + 24
(%i8)  "*************************************************"$
(%i12)  p2:(x+3);
(%o12)   x + 3
(%i13)  gcd(p1,p2);
(%o13)   1
(%i14)  "*************************************************"$
(%i15)  p2:(x-3);
(%o15)   x - 3
(%i16)  gcd(p1,p2);
(%o16)   x - 3
(%i17)  "*************************************************"$
(%i18)  p2:x^2-8*x+15;
(%o18)   x^2 - 8 x + 15
(%i19)  gcd(p1,p2);
(%o19)   x - 3
(%i20)  "*************************************************"$
(%i21)  p2:expand(p2^2);
(%o21)   x^4 - 16 x^3 + 94 x^2 - 240 x + 225
(%i22)  gcd(p1,p2);
(%o22)   x - 3
(%i23)  "*************************************************"$
```

```
(%i24)  divide(p1,p2,x);
(%o24)   [ 1 , 18 x^3 - 107 x^2 + 226 x - 201 ]
(%i25)  divide(p2,p1,x);
(%o25)   [ 1 , - 18 x^3 + 107 x^2 - 226 x + 201 ]
(%i26)  "**************************************************"$
(%i27)  p1:expand((x-1)*(x-2)*(x-3)*(x-4));
(%o27)   x^4 - 10 x^3 + 35 x^2 - 50 x + 24
(%i28)  p2:expand((x+1)*(x-2)*(x+3)*(x-4));
(%o28)   x^4 - 2 x^3 - 13 x^2 + 14 x + 24
(%i29)  p3:expand((x+1)*(x+2)*(x+3)*(x+4));
(%o29)   x^4 + 10 x^3 + 35 x^2 + 50 x + 24
(%i30)  gcd(p1,p2);
(%o30)   x^2 - 6 x + 8
(%i31)  gcd(p1,p3);
(%o31)   1
(%i32)  gcd(p2,p3);
(%o32)   x^2 + 4 x + 3
(%i33)  "**************************************************"$
(%i34)
```

```
(%i1)  x1:(-b+sqrt(b^2-4*a*c))/(2*a);
```

$$(\%o1)\quad \frac{\sqrt{b^2 - 4\,a\,c} - b}{2\,a}$$

```
(%i2)  x2:(-b-sqrt(b^2-4*a*c))/(2*a);
```

$$(\%o2)\quad \frac{-\sqrt{b^2 - 4\,a\,c} - b}{2\,a}$$

```
(%i3)  g(x):=a*x^2+b*x+c;
```

$$(\%o3)\quad g(x) := a\,x^2 + b\,x + c$$

```
(%i4)  g(x1);
```

$$(\%o4)\quad \frac{\left(\sqrt{b^2 - 4\,a\,c} - b\right)^2}{4\,a} + \frac{b\left(\sqrt{b^2 - 4\,a\,c} - b\right)}{2\,a} + c$$

```
(%i5)  expand(%);
```

$$(\%o5)\quad 0$$

```
(%i6)  g(x2);
```

$$(\%o6)\quad \frac{\left(-\sqrt{b^2 - 4\,a\,c} - b\right)^2}{4\,a} + \frac{b\left(-\sqrt{b^2 - 4\,a\,c} - b\right)}{2\,a} + c$$

```
(%i7)  expand(%);
```

$$(\%o7)\quad 0$$

```
(%i8)  "*****************************************************"$
(%i9)  " Beweis der Mitternachtsformel                       "$
(%i10)  "*****************************************************"$
(%i11)
```

```
(%i1)  "*****************************************************"$
(%i2)  " Einige Bruchgleichungen                               "$
(%i3)  "*****************************************************"$
(%i4)  g1:(x-3)/(x+3)+(x+3)/(x-3)=26/(x^2-9);
```

$$(\%o4)\quad \frac{x+3}{x-3}+\frac{x-3}{x+3}=\frac{26}{x^2-9}$$

```
(%i5)  g2:(x+5)/(x-5)+(x-5)/(x+5)+13/6=0;
```

$$(\%o5)\quad \frac{x+5}{x-5}+\frac{x-5}{x+5}+\frac{13}{6}=0$$

```
(%i6)  g3:1/(x^2-9)+(2*x+3)/(x+3)=(3*x+4)/(x-3);
```

$$(\%o6)\quad \frac{1}{x^2-9}+\frac{2x+3}{x+3}=\frac{3x+4}{x-3}$$

```
(%i7)  g4:(5*x+3)/(5*x-3)+(5*x-3)/(5*x+3)=468/(25*x^2-9);
```

$$(\%o7)\quad \frac{5x+3}{5x-3}+\frac{5x-3}{5x+3}=\frac{468}{25x^2-9}$$

```
(%i8)  "*****************************************************"$
(%i9)  solve(g1,x);
```

$$(\%o9)\quad [x=-2,\ x=2]$$

```
(%i10)  "*****************************************************"$
(%i11)  solve(g2,x);
```

$$(\%o11)\quad [x=-1,\ x=1]$$

```
(%i12)  "*****************************************************"$
(%i13)  solve(g3,x);
```

$$(\%o13)\quad [x=-2\sqrt{11}-8,\ x=2\sqrt{11}-8]$$

```
(%i14)  "*****************************************************"$
(%i15)  solve(g4,x);
```

$$(\%o15)\quad [x=-3,\ x=3]$$

```
(%i16)  "*****************************************************"$
(%i17)
```

```
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```

(%i1)
```
"*****************************************************"$
```

(%i2)
```
" Beispiele für Bruchgleichungen "$
```

(%i3)
```
"*****************************************************"$
```

(%i4)
```
g1:3/x-1=0;
```

(%o4)
$$\frac{3}{x} - 1 = 0$$

(%i6)
```
g2:4/(x-1)-1=0;
```

(%o6)
$$\frac{4}{x - 1} - 1 = 0$$

(%i7)
```
g3:a/(x-1)=a;
```

(%o7)
$$\frac{a}{x - 1} = a$$

(%i8)
```
g4:a/x-1=0;
```

(%o8)
$$\frac{a}{x} - 1 = 0$$

(%i9)
```
"*****************************************************"$
```

(%i10)
```
" diese Gleichungen sind gegeben "$
```

(%i11)
```
"*****************************************************"$
```

(%i12)
```
solve(g1,x);
```

(%o12)
$$[x = 3]$$

(%i13)
```
solve(g2,x);
```

(%o13)
$$[x = 5]$$

(%i14)
```
solve(g3,x);
```

(%o14)
$$[x = 2]$$

(%i15)
```
solve(g4,x);
```

(%o15)
$$[x = a]$$

(%i16)
```
"*****************************************************"$
```

(%i17)
```
" das sind die Lösungen "$
```

(%i18)
```
"*****************************************************"$
```

(%i19)
```
Einfache Bruchgleichungen können mit "solve" gut gelöst werden
```

```
(%i1)  "*****************************************************"$
(%i2)  " Gleichungssysteme, die man üblicherweise durch     "$
(%i3)  " Substitution löst                                  "$
(%i4)  "*****************************************************"$
(%i5)  g1:3/(x+1)-2/(y+1)=5/7;
```

$$(\%o5)\quad \frac{3}{x+1}-\frac{2}{y+1}=\frac{5}{7}$$

```
(%i6)  g2:5/(x+1)-7/(y+1)=2/3;
```

$$(\%o6)\quad \frac{5}{x+1}-\frac{7}{y+1}=\frac{2}{3}$$

Warum behandelt man solche Gleichungen überhaupt???

Eventueller Hintergrund: Mustererkennung

```
(%i7)  "*****************************************************"$
(%i8)  solve([g1,g2],[x,y]);
(%o8)   [ [ x = - 1 , y = - 1 ] , [ x = 2 , y = 6 ] ]
(%i9)  "*****************************************************"$
(%i10)  g1:7/(3*x+2)+3/(2*y+1)=12/5;
```

$$(\%o10)\quad \frac{3}{2\,y+1}+\frac{7}{3\,x+2}=\frac{12}{5}$$

```
(%i11)  g2:5/(3*x+2)+9/(2*y+1)=4;
```

$$(\%o11)\quad \frac{9}{2\,y+1}+\frac{5}{3\,x+2}=4$$

```
(%i12)  "*****************************************************"$
(%i13)  solve([g1,g2],[x,y]);
```

$$(\%o13)\quad \left[\left[x=-\frac{2}{3},\ y=-\frac{1}{2}\right],\ [x=1,\ y=1]\right]$$

```
(%i14)  "*****************************************************"$
(%i15)  g1:10/(4*x+2*y+5)+20/(6*x+3*y+5)=7/12;
```

$$(\%o15)\quad \frac{20}{3\,y+6\,x+5}+\frac{10}{2\,y+4\,x+5}=\frac{7}{12}$$

```
(%i16)  g2:20/(4*x+2*y+5)+30/(6*x+3*y+5)=1;
```

$$(\%o16)\quad \frac{30}{3\,y+6\,x+5}+\frac{20}{2\,y+4\,x+5}=1$$

```
(%i17)  "*****************************************************"$
(%i18)  solve([g1,g2],[x,y]);
(%o18)   [ ]
(%i19)  "*****************************************************"$
(%i20)  g1:10*u+20*v=7/12;
```

$$(\%o20)\quad 20\,v+10\,u=\frac{7}{12}$$

```
(%i21)  g2:20*u+30*v=1;
```

$$(\%o21)\quad 30\,v+20\,u=1$$

Linearisierung

```
(%i22)  "*****************************************************"$
```

```
(%i23)  solve([g1,g2],[u,v]);
```

$$(\%o23)\quad \left[\left[u = \frac{1}{40}, v = \frac{1}{60} \right]\right]$$

```
(%i24)  g1:4*x+2*y+5=40;
```

$$(\%o24)\quad 2\,y + 4\,x + 5 = 40$$

```
(%i25)  g2:6*x+3*y+5=60;
```

$$(\%o25)\quad 3\,y + 6\,x + 5 = 60$$

```
(%i26)  solve([g1,g2],[x,y]);
Inconsistent equations:  (2)
 -- an error.  Quitting.  To debug this try DEBUGMODE(TRUE);
(%i27)  A:coefmatrix([g1,g2],[x,y]);
```

$$(\%o27)\quad \begin{bmatrix} 4 & 2 \\ 6 & 3 \end{bmatrix}$$

```
(%i28)  invert(A);
Division by 0
 -- an error.  Quitting.  To debug this try DEBUGMODE(TRUE);
(%i29)  determinant(A);
```

$$(\%o29)\quad 0$$

```
(%i30)  "*****************************************************"$
(%i31)  " Das System ist nicht lösbar                         "$
(%i32)  "*****************************************************"$
(%i33)  g1:17/(9*x+2*y+5)+1/(3*x-2*y+8)=21/20;
```

$$(\%o33)\quad \frac{17}{2\,y + 9\,x + 5} + \frac{1}{-2\,y + 3\,x + 8} = \frac{21}{20}$$

```
(%i34)  g2:13/(9*x+2*y+5)+5/(3*x-2*y+8)=33/20;
```

$$(\%o34)\quad \frac{13}{2\,y + 9\,x + 5} + \frac{5}{-2\,y + 3\,x + 8} = \frac{33}{20}$$

```
(%i35)  "*****************************************************"$
(%i36)  solve([g1,g2],[x,y]);
```

$$(\%o36)\quad \left[\left[x = -\frac{13}{12}, y = \frac{19}{8} \right], \left[x = 1 , y = 3 \right]\right]$$

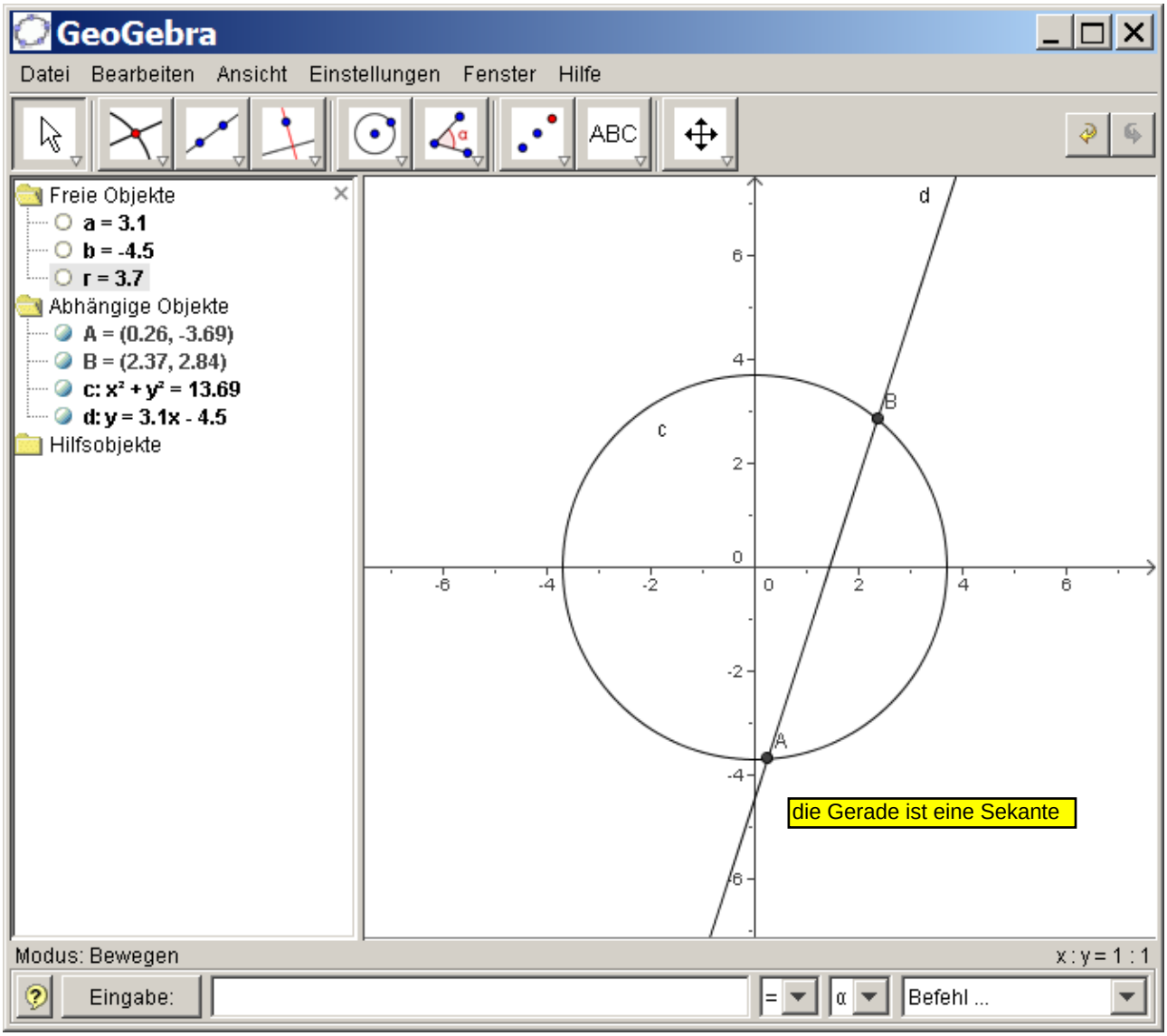
```
(%i37)  "*****************************************************"$
(%i38)
```

```
(%i1)  "*****************************************************"$
(%i2)  " Gleichungssysteme lösen                             "$
(%i3)  "*****************************************************"$
(%i4)  g1:x+y=5;
(%o4)   y + x = 5
(%i5)  g2:x-y=-1;
(%o5)   x - y = - 1
(%i6)  "*****************************************************"$
(%i7)  solve([g1,g2],[x,y]);
(%o7)   [ [ x = 2 , y = 3 ] ]
(%i8)  "*****************************************************"$
(%i9)  A:coefmatrix([g1,g2],[x,y]);
(%o9)   [ 1   1 ]
        [       ]
        [ 1  - 1 ]
(%i10)  b:matrix([5], [-1]);
(%o10)  [  5 ]
        [    ]
        [ - 1 ]
(%i11)  invert(A).b;
(%o11)  [ 2 ]
        [   ]
        [ 3 ]
(%i12)  "*****************************************************"$
(%i13)  " Matrizenmethode                                    "$
(%i14)  "*****************************************************"$
(%i15)  g1;
(%o15)   y + x = 5
(%i16)  g2;
(%o16)   x - y = - 1
(%i17)  g:g1+g2;
(%o17)   2 x = 4
(%i18)  g:g/2;
(%o18)   x = 2
(%i19)  g1,x=2;
(%o19)   y + 2 = 5
(%i20)  g:%-2;
(%o20)   y = 3
(%i21)  "*****************************************************"$
(%i22)  " Eliminationsverfahren                              "$
```

```
(%i23)  "*****************************************************"$
(%i24)  g1;
(%o24)   y + x = 5
(%i25)  g2;
(%o25)   x - y = - 1
(%i26)  g3:g1-y;
(%o26)   x = 5 - y
(%i27)  g4:g2+y;
(%o27)   x = y - 1
(%i29)  g:5-y=y-1;
(%o29)   5 - y = y - 1
(%i30)  g:g+y;
(%o30)   5 = 2 y - 1
(%i31)  g:g+1;
(%o31)   6 = 2 y
(%i32)  g:g/2;
(%o32)   3 = y
(%i33)  g1,y=3;
(%o33)   x + 3 = 5
(%i34)  g:%-3;
(%o34)   x = 2
(%i35)  "*****************************************************"$
(%i36)  " Gleichsetzungsverfahren                              "$
(%i37)  "*****************************************************"$
(%i38)  g1;
(%o38)   y + x = 5
(%i39)  g2;
(%o39)   x - y = - 1
(%i40)  g:g1-x;
(%o40)   y = 5 - x
(%i41)  g2,y=5-x;
(%o41)   2 x - 5 = - 1
(%i42)  g:%+5;
(%o42)   2 x = 4
(%i43)  g:g/2;
(%o43)   x = 2
(%i44)  "*****************************************************"$
(%i45)  " Substitutionsverfahren                               "$
```

Schnittpunkte von Kreis und Gerade, Seite 1 von 2, Johann Weilharter HAK/HAS(HIT) Tamsweg

Schnittpunkt von Kreis und Geraden



Schnittpunkte von Kreis und Gerade, Seite 2 von 2, Johann Weilharter HAK/HAS(HIT) Tamsweg

Die Lösung mit Maxima (Ermittlung der Schnittpunkte)

```
(%i1)  kreis:x^2+y^2=13.69;
```

(%o1) $y^2 + x^2 = 13.69$

```
(%i2)  gerade:y=3.1*x-4.5;
```

(%o2) $y = 3.1000000000000001\,x - 4.5$

```
(%i3)  solve([kreis,gerade],[x,y]);
RAT replaced -13.69 by -1369//100 = -13.69
RAT replaced 4.5 by 9//2 = 4.5
RAT replaced -3.1 by -31//10 = -3.1
```

(%o3) $$\left[\left[x = -\frac{\sqrt{1250009}-1395}{1061}, y = -\frac{31\sqrt{1250009}+4500}{10610}\right], \left[x = \frac{\sqrt{1250009}+1395}{1061}, y = \frac{31\sqrt{1250009}-4500}{10610}\right]\right]$$

```
(%i4)  %,numer;
```

(%o4) [[x = 0.26103862991045296 , y = - 3.6907802472775959] , [x = 2.3685560920499613 , y = 2.8425238853548813]]

```
(%i5)
```

```
wxMaxima 0.6.4 http://wxmaxima.sourceforge.net
Maxima 5.9.1 http://maxima.sourceforge.net
Using Lisp Kyoto Common Lisp GCL 2.6.5 (aka GCL)
Distributed under the GNU Public License. See the file COPYING.
Dedicated to the memory of William Schelter.
This is a development version of Maxima. The function bug_report()
provides bug reporting information.
(%i1)  "Lösung von kubischen Gleichungen"$
(%i2)  "*************************************************"$
(%i3)  g1:x^3-6*x+13=0;
(%o3)   x^3 - 6 x + 13 = 0
(%i4)  g2:x^3+6*x^2+13*x=0;
(%o4)   x^3 + 6 x^2 + 13 x = 0
(%i5)  g3:x^3+6*x^2-13=0;
(%o5)   x^3 + 6 x^2 - 13 = 0
(%i6)  "*************************************************"$
(%i7)  solve(g1,x);
(%o7)   [ x = 2 (sqrt(3) %i/2 - 1/2) / (sqrt(137)/2 - 13/2)^(1/3) + (sqrt(137)/2 - 13/2)^(1/3) (- sqrt(3) %i/2 - 1/2) , x = (sqrt(137)/2 - 13/2)^(1/3) (sqrt(3) %i/2 - 1/2) + 2 (- sqrt(3) %i/2 - 1/2) / (sqrt(137)/2 - 13/2)^(1/3) , x = (sqrt(137)/2 - 13/2)^(1/3) + 2/(sqrt(137)/2 - 13/2)^(1/3) ]
(%i8)  %,numer;
(%o8)   [ x = - 2.3116204564982792 (0.8660254037844386 %i - 0.5) - 0.86519393543941359 (- 0.8660254037844386 %i - 0.5) , x = - 0.86519393543941359 (0.8660254037844386 %i - 0.5) - 2.3116204564982792 (- 0.8660254037844386 %i - 0.5) , x = - 3.1768143919376928 ]
(%i9)  "*************************************************"$
(%i10)  solve(g2,x);
(%o10)   [ x = - 2 %i - 3 , x = 2 %i - 3 , x = 0 ]
(%i11)  "*************************************************"$
(%i12)  solve(g3,x);
(%o12)   [ x = (- sqrt(3) %i/2 - 1/2) (sqrt(247) %i/2 - 3/2)^(1/3) + 4 (sqrt(3) %i/2 - 1/2) / (sqrt(247) %i/2 - 3/2)^(1/3) - 2 , x = (sqrt(3) %i/2 - 1/2)
```

$$\left(\frac{\sqrt{247}\ \%i}{2}-\frac{3}{2}\right)^{1/3}+\frac{4\left(-\frac{\sqrt{3}\ \%i}{2}-\frac{1}{2}\right)}{\left(\frac{\sqrt{247}\ \%i}{2}-\frac{3}{2}\right)^{1/3}}-2\ ,\ x=\left(\frac{\sqrt{247}\ \%i}{2}-\frac{3}{2}\right)^{1/3}+\frac{4}{\left(\frac{\sqrt{247}\ \%i}{2}-\frac{3}{2}\right)^{1/3}}-2\ ]$$

```
(%i13)  %,numer;
```

$$[\ x=\left(-0.8660254037844386\ \%i-0.5\right)\left(7.8581168227508558\ \%i-1.5\right)^{0.3333333333333331}+\frac{4\left(0.8660254037844386\ \%i-0.5\right)}{\left(7.8581168227508558\ \%i-1.5\right)^{0.3333333333333331}}-2\ ,\ x=\left(0.8660254037844386\ \%i-0.5\right)\left(7.8581168227508558\ \%i-1.5\right)^{0.3333333333333331}+\frac{4\left(-0.8660254037844386\ \%i-0.5\right)}{\left(7.8581168227508558\ \%i-1.5\right)^{0.3333333333333331}}-2\ ,\ x=\left(7.8581168227508558\ \%i-1.5\right)^{0.3333333333333331}+\frac{4}{\left(7.8581168227508558\ \%i-1.5\right)^{0.3333333333333331}}-2\ ]$$

```
(%i14)  "*****************************************************"$
(%i15)  " Manchmal ist die Verwendung von ALLROOTS besser"$
(%i16)  "*****************************************************"$
(%i17)  allroots(g1);
(%o17)   [ x = 1.2526421119445246 %i + 1.5884071959688464 , x =
1.5884071959688464 - 1.2526421119445246 %i , x = - 3.1768143919376928 ]
(%i18)  allroots(g2);
(%o18)   [ x = 0.0 , x = 1.9999999999999996 %i - 3.0 , x = -
1.9999999999999996 %i - 3.0 ]
(%i19)  allroots(g3);
(%o19)   [ x = 1.33159580731342 , x = - 1.7486771373723504 , x = -
5.5829186699410691 ]
(%i20)  "*****************************************************"$
(%i21)  " Die Lösungen sind übersichtlicher "$
(%i22)  "*****************************************************"$
(%i23)  Anmerkung: vielleicht hat das mit dem bekannten Fehler von WXMAXIMA zu tun
```

```
wxMaxima 0.6.4 http://wxmaxima.sourceforge.net
Maxima 5.9.1 http://maxima.sourceforge.net
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This is a development version of Maxima. The function bug_report()
provides bug reporting information.
(%i1)  " Lösung von Polynomgleichungen mit ALLROOTS "$
(%i2)  "*************************************************"$
(%i3)  g1:x^3=4*x^2+5*x;
(%o3)   x^3 = 4 x^2 + 5 x
(%i4)  g2:x^3+10*x=7*x^2;
(%o4)   x^3 + 10 x = 7 x^2
(%i5)  g3:4*x^3-2*x^2-5*x=0;
(%o5)   4 x^3 - 2 x^2 - 5 x = 0
(%i6)  g4:x^4-12*x^3+35*x^2=0;
(%o6)   x^4 - 12 x^3 + 35 x^2 = 0
(%i7)  g5:x^4+x^3+x^2=0;
(%o7)   x^4 + x^3 + x^2 = 0
(%i8)  g6:x^3+2*x^2-2*x-1=0;
(%o8)   x^3 + 2 x^2 - 2 x - 1 = 0
(%i9)  "*************************************************"$
(%i10)  " Diese Gleichungen sind gegeben "$
(%i11)  "*************************************************"$
(%i12)  allroots(g1);
(%o12)   [ x = 0.0 , x = - 1.0 , x = 5.0 ]
(%i13)  allroots(g2);
(%o13)   [ x = 0.0 , x = 2.0 , x = 5.0 ]
(%i14)  allroots(g3);
(%o14)   [ x = 0.0 , x = - 0.89564392373895985 , x = 1.3956439237389602 ]
(%i15)  allroots(g4);
(%o15)   [ x = 0.0 , x = 0.0 , x = 4.9999999999999991 , x = 7.0000000000000018
]
(%i16)  allroots(g5);
(%o16)   [ x = 0.0 , x = 0.0 , x = 0.8660254037844386 %i - 0.5 , x = -
0.8660254037844386 %i - 0.5 ]
(%i17)  allroots(g6);
(%o17)   [ x = - 0.38196601125010365 , x = 0.99999999999999911 , x = -
2.6180339887498953 ]
```

```
(%i18)  "*************************************************"$
(%i19)  " Das sind die Lösungen "$
(%i20)  "*************************************************"$
(%i21)
```

```
(%i1)  "Näherungsverfahren"$
(%i2)  f(x):=x^2-8*x+12;
(%o2)  f(x) := x^2 - 8 x + 12
(%i3)  ab:diff(f(x),x);
(%o3)  2 x - 8
(%i4)  a(x):=2*x-8;
(%o4)  a(x) := 2 x - 8
(%i5)  "*****************************************************"$
(%i6)  x[0]:7;
(%o6)  7
(%i7)  g(n):=x[n+1]:x[n]-f(x[n])/a(x[n]);
                                                    f(x_n)
(%o7)  g(n) := x_(n+1) : x_n - ------
                                                    a(x_n)
(%i8)  "*****************************************************"$
(%i9)  g(0),numer;
(%o9)  6.166666666666667
(%i10) g(1);
(%o10)  6.0064102564102573
(%i11) g(2);
(%o11)  6.0000102400262145
(%i12) g(3);
(%o12)  6.0000000000262146
(%i13) "*****************************************************"$
(%i14) x[0]:-5;
(%o14)  - 5
(%i15) g(0),numer;
(%o15)  - 0.72222222222222232
(%i16) g(1);
(%o16)  1.2153594771241829
(%i17) g(2);
(%o17)  1.8894541781818543
(%i18) g(3);
(%o18)  1.9971049245661672
(%i19) g(4);
(%o19)  1.9999979076632943
(%i20) g(5);
(%o20)  1.9999999999989053
(%i21) g(6);
```

Newtonsche Näherung

```
(%o21)  2.0
(%i22)  g(7);
(%o22)  2.0
(%i23)  "**************************************************"$
(%i24)
```

```
(%i1)  "*****************************************************"$
(%i2)  " Nichtlineares Gleichungssystem                      "$
(%i3)  "*****************************************************"$
(%i4)  g1:1/x-1/y=1/36;
```

$$(\%o4)\quad \frac{1}{x}-\frac{1}{y}=\frac{1}{36}$$

```
(%i5)  g2:x*y^2-x^2*y=324;
```

$$(\%o5)\quad x\,y^2-x^2\,y=324$$

```
(%i6)  "*****************************************************"$
(%i7)  solve([g1,g2],[x,y]);
```

$$(\%o7)\quad \left[\left[x=\frac{72}{\sqrt{47}\ \%i+1},\ y=-\frac{3\sqrt{47}\ \%i+3}{2}\right],\ \left[x=-\frac{72}{\sqrt{47}\ \%i-1},\ y=\frac{3\sqrt{47}\ \%i-3}{2}\right],\ [x=9,\ y=12],\ [x=-12,\ y=-9]\right]$$

```
(%i8)  "*****************************************************"$
(%i9)  " Das sind die Lösungen                               "$
(%i10)  "*****************************************************"$
(%i11)
```

```
wxMaxima 0.6.4 http://wxmaxima.sourceforge.net
Maxima 5.9.1 http://maxima.sourceforge.net
Using Lisp Kyoto Common Lisp GCL 2.6.5 (aka GCL)
Distributed under the GNU Public License. See the file COPYING.
Dedicated to the memory of William Schelter.
This is a development version of Maxima. The function bug_report()
provides bug reporting information.
(%i1)  "*****************************************************"$
(%i2)  " Lösen von Polynomgleichungen "$
(%i3)  "*****************************************************"$
(%i4)  a:x^2-8*x+15;
(%o4)   x^2 - 8 x + 15
(%i5)  allroots(a);
(%o5)   [ x = 3.0 , x = 5.0 ]
(%i6)  "*****************************************************"$
(%i7)  b:a*(x+4);
(%o7)   (x + 4) (x^2 - 8 x + 15)
(%i8)  expand(%);
(%o8)   x^3 - 4 x^2 - 17 x + 60
(%i9)  allroots(%);
(%o9)   [ x = 3.0 , x = - 4.0 , x = 5.0 ]
(%i10)  "*****************************************************"$
(%i11)  c:b*(x-10);
(%o11)   (x - 10) (x + 4) (x^2 - 8 x + 15)
(%i12)  expand(%);
(%o12)   x^4 - 14 x^3 + 23 x^2 + 230 x - 600
(%i13)  allroots(%);
(%o13)   [ x = 3.0000000000000004 , x = - 4.0 , x = 4.9999999999999982 , x =
10.000000000000002 ]
(%i14)  "*****************************************************"$
(%i15)  " ALLROOTS ist manchmal ungenau "$
(%i16)  "*****************************************************"$
(%i17)
```

```
(%i1)  "*************************************************"$
(%i3)  " Bestimme eine parabel aus 3 Punkten             "$
(%i4)  "*************************************************"$
(%i5)  x1:-2.13;
(%o5)   - 2.1299999999999999
(%i6)  y1:6.73;
(%o6)   6.7300000000000004
(%i7)  x2:2.47;
(%o7)   2.4699999999999998
(%i8)  y2:-3.08;
(%o8)   - 3.0800000000000001
(%i9)  x3:4.47;
(%o9)   4.4699999999999998
(%i10)  y3:5.85;
(%o10)   5.8499999999999996
(%i11)  "*************************************************"$
(%i12)  " Das sind die koordinaten der drei Punkte       "$
(%i13)  "*************************************************"$
(%i14)  g(x,y):=y=a*x^2+b*x+c;
                                    2
(%o14)  g(x, y) := y = a x  + b x + c
(%i15)  "*************************************************"$
(%i16)  " Allgemeiner Ansatz                             "$
(%i17)  "*************************************************"$
(%i18)  g1:g(x1,y1);
(%o18)   6.7300000000000004 = c - 2.1299999999999999 b + 4.5368999999999993
a
(%i19)  g2:g(x2,y2);
(%o19)   - 3.0800000000000001 = c + 2.4699999999999998 b +
6.1008999999999984 a
(%i20)  g3:g(x3,y3);
(%o20)   5.8499999999999996 = c + 4.4699999999999998 b + 19.980899999999998
a
(%i21)  "*************************************************"$
(%i22)  " Das sind die Gleichungen                       "$
(%i23)  "*************************************************"$
(%i24)  solve([g1,g2,g3],[a,b,c]);
RAT replaced 6.73 by 673//100 = 6.73
RAT replaced -4.5369 by -21455//4729 = -4.53689997885388
RAT replaced 2.13 by 213//100 = 2.13
```

dieser Anzeigefehler von WXMAXIMA ist bekannt

```
RAT replaced -3.08 by -77//25 = -3.08
RAT replaced -6.1009 by -12879//2111 = -6.10090004737091
RAT replaced -2.47 by -247//100 = -2.47
RAT replaced 5.85 by 117//20 = 5.85
RAT replaced -19.9809 by -61721//3089 = -19.9808999676271
RAT replaced -4.47 by -447//100 = -4.47
```

$$(\%o24)\quad [\ [\ a = \frac{935879299370059}{936218501206000}\ ,\ b = -\frac{231478669242811}{93621850120600}\ ,\ c = -\frac{28757359152772993}{9362185012060000}\ ]\ ]$$

```
(%i25)  %,numer;
(%o25)   [ [ a = 0.99963768945443388 , b = - 2.472485524956292 , c = -
3.0716503803042654 ] ]
(%i26)  y=x^2-2.47*x-3.07;
```

$$(\%o26)\quad y = x^2 - 2.4699999999999998\ x - 3.0699999999999998$$

```
(%i27)  "**************************************************"$
(%i28)  " das ist die gesuchte Parabel                     "$
(%i29)  "**************************************************"$
(%i30)
```

```
(%i1)  g1:x^2+2*x+26=0;
```

$$(\%o1)\quad x^2 + 2\,x + 26 = 0$$

```
(%i2)  g2:x^2+6*x+10=0;
```

$$(\%o2)\quad x^2 + 6\,x + 10 = 0$$

```
(%i3)  g3:4*x^2-4*x+5=0;
```

$$(\%o3)\quad 4\,x^2 - 4\,x + 5 = 0$$

```
(%i4)  g4:4*x^2-4*x+37=0;
```

$$(\%o4)\quad 4\,x^2 - 4\,x + 37 = 0$$

```
(%i5)  g5:x^2-6*x+10=0;
```

$$(\%o5)\quad x^2 - 6\,x + 10 = 0$$

```
(%i6)  g6:x^2-4*x+5=0;
```

$$(\%o6)\quad x^2 - 4\,x + 5 = 0$$

```
(%i7)  g7:4*x^2-2*x+7=0;
```

$$(\%o7)\quad 4\,x^2 - 2\,x + 7 = 0$$

```
(%i8)  g8:8*x^2-12*x+17=0;
```

$$(\%o8)\quad 8\,x^2 - 12\,x + 17 = 0$$

```
(%i9)  g9:x^2+18*x+97=0;
```

$$(\%o9)\quad x^2 + 18\,x + 97 = 0$$

```
(%i10)  g10:x^2+x+1=0;
```

$$(\%o10)\quad x^2 + x + 1 = 0$$

```
(%i11)  g11:4*x^2-12*x+15=0;
```

$$(\%o11)\quad 4\,x^2 - 12\,x + 15 = 0$$

```
(%i12)  g12:16*x^2-64*x+89=0;
```

$$(\%o12)\quad 16\,x^2 - 64\,x + 89 = 0$$

```
(%i13)  solve(g1,x);
```

$$(\%o13)\quad [\,x = -5\,\%i - 1\,,\ x = 5\,\%i - 1\,]$$

```
(%i14)  solve(g2,x);
```

$$(\%o14)\quad [\,x = -\%i - 3\,,\ x = \%i - 3\,]$$

```
(%i15)  solve(g3,x);
```

$$(\%o15)\quad \left[\,x = -\frac{2\,\%i - 1}{2}\,,\ x = \frac{2\,\%i + 1}{2}\,\right]$$

```
(%i16)  solve(g4,x);
```

$$(\%o16)\quad \left[\,x = -\frac{6\,\%i - 1}{2}\,,\ x = \frac{6\,\%i + 1}{2}\,\right]$$

```
(%i17)  solve(g5,x);
```

$$(\%o17)\quad [\,x = 3 - \%i\,,\ x = \%i + 3\,]$$

```
(%i18)  solve(g6,x);
```

```
(%o18)   [ x = 2 - %i , x = %i + 2 ]
(%i19)  solve(g7,x);
```

$$(\%o19)\quad \left[x = -\frac{3\sqrt{3}\ \%i - 1}{4}, x = \frac{3\sqrt{3}\ \%i + 1}{4} \right]$$

```
(%i20)  solve(g8,x);
```

$$(\%o20)\quad \left[x = -\frac{5\ \%i - 3}{4}, x = \frac{5\ \%i + 3}{4} \right]$$

```
(%i21)  solve(g9,x);
(%o21)   [ x = - 4 %i - 9 , x = 4 %i - 9 ]
(%i22)  solve(g10,x);
```

$$(\%o22)\quad \left[x = -\frac{\sqrt{3}\ \%i + 1}{2}, x = \frac{\sqrt{3}\ \%i - 1}{2} \right]$$

```
(%i23)  solve(g11,x);
```

$$(\%o23)\quad \left[x = -\frac{\sqrt{6}\ \%i - 3}{2}, x = \frac{\sqrt{6}\ \%i + 3}{2} \right]$$

```
(%i24)  solve(g12,x);
```

$$(\%o24)\quad \left[x = -\frac{5\ \%i - 8}{4}, x = \frac{5\ \%i + 8}{4} \right]$$

```
(%i25)  "*****************************************************"$
(%i26)  " Alle Gleichungen haben komplexe Lösungen           "$
(%i27)  "*****************************************************"$
(%i28)
```

```
(%i1)  "*****************************************************"$
(%i2)  " Einfache Wurzelgleichungen                          "$
(%i3)  "*****************************************************"$
(%i4)  g1:sqrt(2*x)=8;
(%o4)  sqrt(2) sqrt(x) = 8
(%i5)  solve(g1,x);
(%o5)  [ x = 32 ]
(%i6)  "*****************************************************"$
(%i7)  g2:sqrt(6*x)-5=1;
(%o7)  sqrt(6) sqrt(x) - 5 = 1
(%i8)  solve(g2,x);
(%o8)  [ x = 6 ]
(%i9)  "*****************************************************"$
(%i10)  g3:4*sqrt(3*x)-11=1;
(%o10)  4 sqrt(3) sqrt(x) - 11 = 1
(%i11)  solve(g3,x);
(%o11)  [ x = 3 ]
(%i12)  "*****************************************************"$
(%i13)  g4:sqrt(x-2)=1;
(%o13)  sqrt(x - 2) = 1
(%i14)  solve(g4,x);
(%o14)  [ x = 3 ]
(%i15)  "*****************************************************"$
(%i16)  g5:20=sqrt(15-x);
(%o16)  20 = sqrt(15 - x)
(%i17)  solve(g5,x);
(%o17)  [ x = - 385 ]
(%i18)  "*****************************************************"$
(%i19)  g6:4*sqrt(x+3)-15=5;
(%o19)  4 sqrt(x + 3) - 15 = 5
(%i20)  solve(g6,x);
(%o20)  [ x = 22 ]
(%i21)  "*****************************************************"$
(%i22)  g7:g7:5*sqrt(3*x+1)=3*sqrt(5*x+25);
(%o22)  5 sqrt(3 x + 1) = 3 sqrt(5 x + 25)
(%i23)  solve(g7,x);
(%o23)  [sqrt(5 x + 25) = (5 sqrt(3 x + 1))/3]
```

```
(%i24)  g7:g7^2;
(%o24)   25 (3 x + 1) = 9 (5 x + 25)
(%i25)  solve(%,x);
(%o25)   [ x = 20/3 ]
(%i26)  "*****************************************************"$
(%i27)  " Quadrieren hilft                                    "$
(%i28)  "*****************************************************"$
(%i29)  g8:4*sqrt(4*x+1)=3*sqrt(7*x+2);
(%o29)   4 sqrt(4 x + 1) = 3 sqrt(7 x + 2)
(%i30)  solve(g8,x);
(%o30)   [sqrt(7 x + 2) = 4 sqrt(4 x + 1)/3 ]
(%i31)  g8:g8^2;
(%o31)   16 (4 x + 1) = 9 (7 x + 2)
(%i32)  solve(g8,x);
(%o32)   [ x = 2 ]
(%i33)  "*****************************************************"$
(%i34)  g9:5*sqrt(x)-1=7*sqrt(x)-5;
(%o34)   5 sqrt(x) - 1 = 7 sqrt(x) - 5
(%i35)  solve(g9,x);
(%o35)   [ x = 4 ]
(%i36)  "*****************************************************"$
(%i37)  g10:7*sqrt(x)+9=6*(3*sqrt(x)-4);
(%o37)   7 sqrt(x) + 9 = 6 (3 sqrt(x) - 4)
(%i38)  solve(g10,x);
(%o38)   [ x = 9 ]
(%i39)  "*****************************************************"$
(%i40)  g11:sqrt(x+3)-sqrt(x)=1;
(%o40)   sqrt(x + 3) - sqrt(x) = 1
(%i41)  solve(g11,x);
(%o41)   [sqrt(x) = sqrt(x + 3) - 1 ]
(%i42)  g11:g11^2;
(%o42)   (sqrt(x + 3) - sqrt(x))^2 = 1
(%i43)  g11:expand(g11);
(%o43)   - 2 sqrt(x) sqrt(x + 3) + 2 x + 3 = 1
(%i44)  g11:g11-2*x-3;
```

```
(%o44)  - 2 sqrt(x) sqrt(x + 3) = - 2 x - 2
(%i45)  g11:g11^2;
(%o45)  4 x (x + 3) = (- 2 x - 2)^2
(%i46)  solve(g11,x);
(%o46)  [ x = 1 ]
(%i47)  "******************************************************"$
(%i48)  g12:sqrt(x+14)+sqrt(x+7)=7;
(%o48)  sqrt(x + 14) + sqrt(x + 7) = 7
(%i49)  g12:g12^2;
(%o49)  (sqrt(x + 14) + sqrt(x + 7))^2 = 49
(%i50)  g12:expand(g12);
(%o50)  2 sqrt(x + 7) sqrt(x + 14) + 2 x + 21 = 49
(%i51)  g12:g12-2*x-21;
(%o51)  2 sqrt(x + 7) sqrt(x + 14) = 28 - 2 x
(%i52)  g12:g12^2;
(%o52)  4 (x + 7) (x + 14) = (28 - 2 x)^2
(%i53)  solve(g12,x);
(%o53)  [ x = 2 ]
(%i54)  "******************************************************"$
(%i55)  g13:sqrt(x+9)-sqrt(x-4)=1;
(%o55)  sqrt(x + 9) - sqrt(x - 4) = 1
(%i56)  g13:g13^2;
(%o56)  (sqrt(x + 9) - sqrt(x - 4))^2 = 1
(%i57)  g13:expand(g13);
(%o57)  - 2 sqrt(x - 4) sqrt(x + 9) + 2 x + 5 = 1
(%i58)  g13:g13-2*x-5;
(%o58)  - 2 sqrt(x - 4) sqrt(x + 9) = - 2 x - 4
(%i59)  g13:g13^2;
(%o59)  4 (x - 4) (x + 9) = (- 2 x - 4)^2
(%i60)  solve(g13,x);
(%o60)  [ x = 40 ]
(%i62)  "******************************************************"$
(%i63)  g14:sqrt(x+4)+sqrt(x+11)=sqrt(4*x+29);
(%o63)  sqrt(x + 11) + sqrt(x + 4) = sqrt(4 x + 29)
(%i64)  g14:g14^2;
```

```
(%o64)  (sqrt(x + 11) + sqrt(x + 4))^2 = 4 x + 29
(%i65)  g14:expand(g14);
(%o65)  2 sqrt(x + 4) sqrt(x + 11) + 2 x + 15 = 4 x + 29
(%i66)  g14:g14-2*x-15;
(%o66)  2 sqrt(x + 4) sqrt(x + 11) = 2 x + 14
(%i67)  g14:g14^2;
(%o67)  4 (x + 4) (x + 11) = (2 x + 14)^2
(%i68)  solve(g14,x);
(%o68)  [ x = 5 ]
(%i69)  "*****************************************************"$
(%i70)  g15:sqrt(x+12)-sqrt(x-3)=sqrt(x+32)-sqrt(x+5);
(%o70)  sqrt(x + 12) - sqrt(x - 3) = sqrt(x + 32) - sqrt(x + 5)
(%i71)  g15:g15^2;
(%o71)  (sqrt(x + 12) - sqrt(x - 3))^2 = (sqrt(x + 32) - sqrt(x + 5))^2
(%i72)  g15:expand(g15);
(%o72)  - 2 sqrt(x - 3) sqrt(x + 12) + 2 x + 9 = - 2 sqrt(x + 5) sqrt(x + 32) + 2 x + 37
(%i73)  g15:g15-2*x-9;
(%o73)  - 2 sqrt(x - 3) sqrt(x + 12) = 28 - 2 sqrt(x + 5) sqrt(x + 32)
(%i74)  g15:g15^2;
(%o74)  4 (x - 3) (x + 12) = (28 - 2 sqrt(x + 5) sqrt(x + 32))^2
(%i75)  g15:expand(g15);
(%o75)  4 x^2 + 36 x - 144 = - 112 sqrt(x + 5) sqrt(x + 32) + 4 x^2 + 148 x + 1424
(%i76)  g15:g15-4*x^2-148*x-1424;
(%o76)  - 112 x - 1568 = - 112 sqrt(x + 5) sqrt(x + 32)
(%i77)  g15:g15^2;
(%o77)  (- 112 x - 1568)^2 = 12544 (x + 5) (x + 32)
(%i78)  solve(g15,x);
(%o78)  [ x = 4 ]
(%i79)  "*****************************************************"$
(%i80)
```

```
wxMaxima 0.6.4 http://wxmaxima.sourceforge.net
Maxima 5.9.1 http://maxima.sourceforge.net
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Dedicated to the memory of William Schelter.
This is a development version of Maxima. The function bug_report()
provides bug reporting information.
(%i1)  "*****************************************************"$
(%i2)  " Koeffizientenmatrix "$
(%i3)  "*****************************************************"$
(%i4)  g1:x+y=5;
(%o4)   y + x = 5
(%i5)  g2:x-y=-1;
(%o5)   x - y = - 1
(%i6)  "*****************************************************"$
(%i7)  A:coefmatrix([g1,g2],[x,y]);
                                [ 1    1 ]
(%o7)                           [        ]
                                [ 1  - 1 ]
(%i8)  "*****************************************************"$
(%i9)  g1:x+y+z=3;
(%o9)   z + y + x = 3
(%i10)  g2:2*x-y+3*z=4;
(%o10)   3 z - y + 2 x = 4
(%i11)  g3:-2*x+y-5*z=-6;
(%o11)   - 5 z + y - 2 x = - 6
(%i12)  "*****************************************************"$
(%i13)  B:coefmatrix([g1,g2,g3],[x,y,z]);
                                [  1    1    1 ]
                                [             ]
(%o13)                          [  2  - 1    3 ]
                                [             ]
                                [ - 2    1  - 5 ]
(%i14)  "*****************************************************"$
(%i16)  determinant(A);
(%o16)   - 2
(%i17)  determinant(B);
(%o17)   6
(%i18)  "*****************************************************"$
(%i19)  invert(A);
```

(%o19)
$$\begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{bmatrix}$$

(%i20) `invert(B);`

(%o20)
$$\begin{bmatrix} \frac{1}{3} & 1 & \frac{2}{3} \\ \frac{2}{3} & -\frac{1}{2} & -\frac{1}{6} \\ 0 & -\frac{1}{2} & -\frac{1}{2} \end{bmatrix}$$

(%i21)

```
(%i1)  "*****************************************************"$
(%i2)  " Funktion dritten Grades bestimmen                    "$
(%i3)  "*****************************************************"$
(%i4)  load(vect);
(%o4)
C:/Programme/Maxima-5.9.1/share/maxima/5.9.1/share/vector/vect.mac
(%i5)  x:[10,44,82,102];
(%o5)   [ 10 , 44 , 82 , 102 ]
(%i6)  y:1*x^3+2*x^2+3*x+4;
(%o6)   [ 1234 , 89192 , 565066 , 1082326 ]
(%i7)  kill(all);
(%o0)   DONE
(%i1)  "*****************************************************"$
(%i2)  x1:10;
(%o2)   10
(%i3)  y1:1234;
(%o3)   1234
(%i4)  x2:44;
(%o4)   44
(%i5)  y2:89192;
(%o5)   89192
(%i6)  x3:82;
(%o6)   82
(%i7)  y3:565066;
(%o7)   565066
(%i8)  x4:102;
(%o8)   102
(%i9)  y4:1082326;
(%o9)   1082326
(%i10)  "*****************************************************"$
(%i19)  g(x,y):=y=a*x^3+b*x^2+c*x+d;
                                   3      2
(%o19)   g(x , y) := y = a x  + b x  + c x + d
(%i20)  "*****************************************************"$
(%i21)  g1:g(x1,y1);
(%o21)   1234 = d + 10 c + 100 b + 1000 a
(%i22)  g2:g(x2,y2);
(%o22)   89192 = d + 44 c + 1936 b + 85184 a
(%i23)  g3:g(x3,y3);
```

```
(%o23)   565066 = d + 82 c + 6724 b + 551368 a
(%i24)  g4:g(x4,y4);
(%o24)   1082326 = d + 102 c + 10404 b + 1061208 a
(%i25)  "*****************************************************"$
(%i26)  solve([g1,g2,g3,g4],[a,b,c,d]);
(%o26)   [ [ a = 1 , b = 2 , c = 3 , d = 4 ] ]
(%i27)  y=1*x^3+2*x^2+3*x+4;
                          3      2
(%o27)   y = x  + 2 x  + 3 x + 4
(%i28)  "*****************************************************"$
(%i29)  " Das ist die Lösung                                   "$
(%i30)  "*****************************************************"$
(%i31)
```

```
(%i1)  g1:2*x+6*y+z=9;
(%o1)   z + 6 y + 2 x = 9
(%i2)  g2:3*x-2*y+2*z=3;
(%o2)   2 z - 2 y + 3 x = 3
(%i3)  g3:-x+3*y-z=1;
(%o3)   - z + 3 y - x = 1
(%i4)  A:coefmatrix([g1,g2,g3],[x,y,z]);
```

$$\begin{bmatrix} 2 & 6 & 1 \\ 3 & -2 & 2 \\ -1 & 3 & -1 \end{bmatrix}$$

```
(%i5)  b:matrix([9], [3], [1]);
```

$$\begin{bmatrix} 9 \\ 3 \\ 1 \end{bmatrix}$$

Erzeugung von Spaltenvektoren

```
(%i6)  X:matrix([x1], [x2], [x3]);
```

$$\begin{bmatrix} x1 \\ x2 \\ x3 \end{bmatrix}$$

```
(%i7)  Gleichung:A.X=b;
```

$$\begin{bmatrix} x3 + 6\,x2 + 2\,x1 \\ 2\,x3 - 2\,x2 + 3\,x1 \\ -x3 + 3\,x2 - x1 \end{bmatrix} = \begin{bmatrix} 9 \\ 3 \\ 1 \end{bmatrix}$$

```
(%i9)  "*****************************************************"$
(%i10)  " Lösung über inverse Matrix                          "$
(%i11)  "*****************************************************"$
(%i12)  Loesung:X=invert(A).b;
```

$$\begin{bmatrix} x1 \\ x2 \\ x3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

```
(%i13)
```

```
(%i1)  "*****************************************************"$
(%i2)  " Quadratische Funktion bestimmen                     "$
(%i3)  "*****************************************************"$
(%i4)  load(vect);
(%o4)
C:/Programme/Maxima-5.9.1/share/maxima/5.9.1/share/vector/vect.mac
(%i5)  x:[10,29,64];
(%o5)   [ 10 , 29 , 64 ]
(%i6)  y:3*x-12;
(%o6)   [ 18 , 75 , 180 ]
(%i7)  kill(all);
(%o0)   DONE
(%i1)  "*****************************************************"$
(%i2)  x1:10;
(%o2)   10
(%i3)  y1:18;
(%o3)   18
(%i4)  x2:29;
(%o4)   29
(%i5)  y2:75;
(%o5)   75
(%i6)  x3:64;
(%o6)   64
(%i7)  y3:180;
(%o7)   180
(%i8)  "*****************************************************"$
(%i9)  " Das sind die gegebenen Punkte                       "$
(%i10)  "*****************************************************"$
(%i11)  g(x,y):=y=a*x^2+b*x+c;
(%o11)   g(x , y) := y = a x^2 + b x + c
(%i12)  "*****************************************************"$
(%i13)  g1:g(x1,y1);
(%o13)   18 = c + 10 b + 100 a
(%i15)  g2:g(x2,y2);
(%o15)   75 = c + 29 b + 841 a
(%i16)  g3:g(x3,y3);
(%o16)   180 = c + 64 b + 4096 a
(%i17)  "*****************************************************"$
```

der Aufruf des Unterprogramms ist NICHT notwendig (war ein Versehen :-)

```
(%i18)  solve([g1,g2,g3],[a,b,c]);
(%o18)   [ [ a = 0 , b = 3 , c = - 12 ] ]
(%i19)  "*************************************************"$
(%i20)  " Die drei Punkte liegen auf einer Geraden          "$
(%i21)  "*************************************************"$
(%i22)
```

```
(%i1)  p:1/n;
```

$$(\%o1)\quad \frac{1}{n}$$

```
(%i2)  m:sum(k*p,k,1,n);
```

$$(\%o2)\quad \frac{\sum_{k=1}^{n} k}{n}$$

```
(%i3)  m:m,simpsum;
```

$$(\%o3)\quad \frac{n^2+n}{2\,n}$$

```
(%i4)  m:factor(m);
```

$$(\%o4)\quad \frac{n+1}{2}$$

```
(%i5)  "*****************************************************"$
(%i6)  " Das ist der Erwartungswert (Gleichverteilung)      "$
(%i7)  "*****************************************************"$
(%i8)  v:sum((k-m)^2*p,k,1,n);
```

$$(\%o8)\quad \frac{\sum_{k=1}^{n} \left(k-\frac{n+1}{2}\right)^2}{n}$$

```
(%i9)  v:v,simpsum;
```

$$(\%o9)\quad \frac{\frac{2\,n^3+3\,n^2+n}{6}-\frac{n^3+n^2}{2}+\frac{n^3}{4}-\frac{n^2+n}{2}+\frac{n^2}{2}+\frac{n}{4}}{n}$$

```
(%i10)  v:factor(v);
```

$$(\%o10)\quad \frac{(n-1)(n+1)}{12}$$

```
(%i11)  "*****************************************************"$
(%i12)  " Das ist die Varianz (Gleichverteilung)            "$
(%i13)  "*****************************************************"$
(%i14)  s:sqrt(v);
```

$$(\%o14)\quad \frac{\sqrt{(n-1)(n+1)}}{2\sqrt{3}}$$

```
(%i15)  "*****************************************************"$
(%i16)  " Das ist die Streuung (Gleichverteilung)           "$
(%i17)  "*****************************************************"$
(%i18)
```

```
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(%i1)  "**************************************************"$
(%i2)  integrate(x,x,a,b);
```
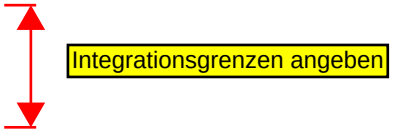

$$(\%o2)\quad \frac{b^2}{2}-\frac{a^2}{2}$$




```
(%i3)  "**************************************************"$
(%i4)  integrate(x^2,x,2,4);
```

$$(\%o4)\quad \frac{56}{3}$$

```
(%i5)  "**************************************************"$
(%i6)  integrate(sin(x),x,0,%pi/2);
(%o6)  1
(%i7)  "**************************************************"$
(%i8)  integrate(abs(x-3),x,2,3);
```

$$(\%o8)\quad \frac{1}{2}$$

```
(%i9)  "**************************************************"$
(%i10)
```

```
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```

(%i1)
```
"*****************************************************"$
```

(%i2)
```
" Bestimmte Integrale "$
```

(%i3)
```
"*****************************************************"$
```

(%i4)
```
f(x):=x^2;
```

(%o4) $$f(x) := x^2$$

(%i5)
```
integrate(f(x),x,0,1);
```

(%o5) $$\frac{1}{3}$$

(%i6)
```
"*****************************************************"$
```

(%i7)
```
f(x);
```

(%o7) $$x^2$$

(%i8)
```
integrate(f(x),x,-1,1);
```

(%o8) $$\frac{2}{3}$$

(%i9)
```
"*****************************************************"$
```

(%i10)
```
f(x);
```

(%o10) $$x^2$$

(%i11)
```
integrate(f(x),x,0,3);
```

(%o11) 9

(%i12)
```
"*****************************************************"$
```

(%i13)
```
f(x):=x+1;
```

(%o13) $$f(x) := x + 1$$

(%i15)
```
integrate(f(x),x,0,5);
```

(%o15) $$\frac{35}{2}$$

(%i16)
```
"*****************************************************"$
```

(%i17)
```
f(x):=x;
```

(%o17) $$f(x) := x$$

(%i18)
```
integrate(f(x),x,0,3);
```

(%o18) $$\frac{9}{2}$$

(%i19)
```
"*****************************************************"$
```

(%i20)

```
wxMaxima 0.6.4 http://wxmaxima.sourceforge.net
Maxima 5.9.1 http://maxima.sourceforge.net
Using Lisp Kyoto Common Lisp GCL 2.6.5 (aka GCL)
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Dedicated to the memory of William Schelter.
This is a development version of Maxima. The function bug_report()
provides bug reporting information.
```

(%i1)
```
"**************************************************"$
```

(%i2)
```
" Unbestimmte Integrale "$
```

(%i3)
```
"**************************************************"$
```

(%i4)
```
f(x):=exp(k*x);
```

(%o4)
$$f(x) := \%e^{k\,x}$$

(%i5)
```
integrate(f(x),x);
```

(%o5)
$$\frac{\%e^{k\,x}}{k}$$

(%i6)
```
"**************************************************"$
```

(%i7)
```
f(x):=cos(3*x-4);
```

(%o7)
$$f(x) := \cos(3\,x - 4)$$

(%i8)
```
integrate(f(x),x);
```

(%o8)
$$\frac{\sin(3\,x - 4)}{3}$$

(%i9)
```
"**************************************************"$
```

(%i10)
```
f(x):=sqrt(1-4*x);
```

(%o10)
$$f(x) := \sqrt{1 - 4\,x}$$

(%i11)
```
integrate(f(x),x);
```

(%o11)
$$-\frac{(1 - 4\,x)^{3/2}}{6}$$

(%i12)
```
"**************************************************"$
```

(%i13)
```
f(x):=x^4*(1+x^5)^(1/3);
```

(%o13)
$$f(x) := x^4\,(1 + x^5)^{1/3}$$

(%i14)
```
integrate(f(x),x);
```

(%o14)
$$\frac{3\,(x^5 + 1)^{4/3}}{20}$$

(%i15)
```
"**************************************************"$
```

(%i16)
```
f(x):=cos(2*x);
```

(%o16)
$$f(x) := \cos(2\,x)$$

(%i17)
```
integrate(f(x),x);
```

(%o17)
$$\frac{\sin(2\,x)}{2}$$

```
(%i18)  "*****************************************************"$
(%i19)  f(x):=(cos(x))^2;
(%o19)  f(x) := cos(x)^2
(%i20)  integrate(f(x),x);
                sin(2 x)
                --------  + x
                   2
(%o20)          ------------
                     2
(%i21)  "*****************************************************"$
(%i22)
```

```
(%i1)  "*****************************************************"$
(%i2)  " Integralrechnung - FREIER FALL                      "$
(%i3)  "*****************************************************"$
(%i4)  a(t):=g;
(%o4)   a(t) := g
(%i5)  "*****************************************************"$
(%i6)  " Beschleunigung ist Erdbeschleunigung                "$
(%i7)  "*****************************************************"$
(%i8)  v(t):=integrate(a(t),t);
(%o8)   v(t) := INTEGRATE(a(t), t)
(%i9)  v(t);
(%o9)   g t
(%i10)  "*****************************************************"$
(%i11)  " Die Geschwindigkeit                                "$
(%i12)  "*****************************************************"$
(%i13)  s(t):=integrate(v(t),t);
(%o13)   s(t) := INTEGRATE(v(t), t)
(%i14)  s(t);
                                 g t^2
(%o14)                           -----
                                   2
(%i15)  "*****************************************************"$
(%i16)  " Das Weg-Zeit-Gesetz                                "$
(%i17)  "*****************************************************"$
(%i18)
```

durch zweimalige Integration kann man das Weg-Zeit-Gesetz des freien Falls bestimmen

```
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(%i1)  "+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++"$
(%i2)  " Bestimme Integrale "$
(%i3)  "+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++"$
(%i4)  f(x):=x;
(%o4)  f(x) := x
(%i5)  integrate(f(x),x,a,b);
                              2    2
                             b    a
(%o5)                        -- - --
                             2    2
(%i6)  "+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++"$
(%i7)  f(x):=x^2;
                              2
(%o7)  f(x) := x
(%i8)  integrate(f(x),x,a,b);
                              3    3
                             b    a
(%o8)                        -- - --
                             3    3
(%i9)  "+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++"$
(%i10)  f(x):=x^3;
                              3
(%o10)  f(x) := x
(%i11)  integrate(f(x),x,a,b);
                              4    4
                             b    a
(%o11)                       -- - --
                             4    4
(%i12)
"+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++"$
(%i13)  f(x):=sin(x);
(%o13)  f(x) := sin(x)
(%i14)  integrate(f(x),x,a,b);
Is  b - a  positive, negative, or zero? positive;
(%o14)  cos(a) - cos(b)
(%i15)
"+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++"$
(%i16)  f(x):=abs(x-3);
(%o16)  f(x) := |x - 3|
(%i17)  integrate(f(x),x,a,b);
```

(%o17) $$\int_a^b \left(\left|x - 3\right|\right)\,dx$$

(%i18)

```
"+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++"$
```

(%i19)
```
f(x):=F(x);
```

(%o19) $$f\left(x\right) := F\left(x\right)$$

(%i20)
```
integrate(F(x),x,a,b);
```

(%o20) $$\int_a^b \left(F\left(x\right)\right)\,dx$$

(%i21)
```
%=F(b)-F(a);
```

(%o21) $$\int_a^b \left(F\left(x\right)\right)\,dx = F\left(b\right) - F\left(a\right)$$

(%i23)

```
"+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++"$
```

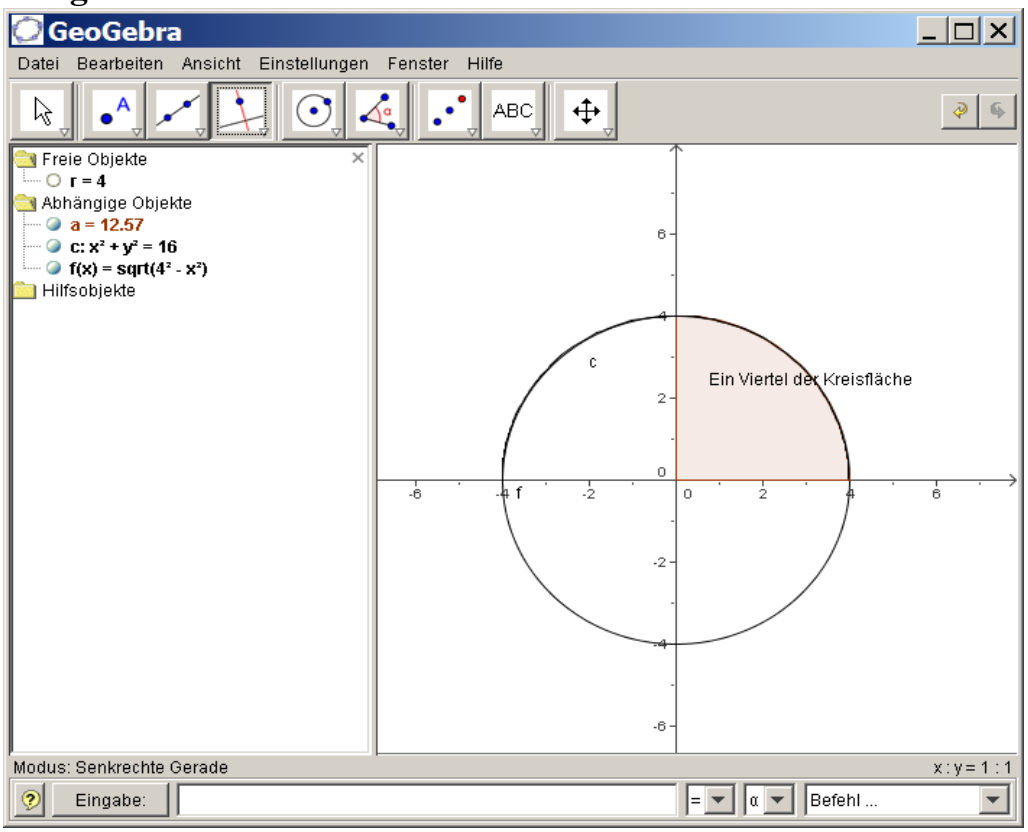
(%i24)
```
" Das ist der Hauptsatz der Integralrechnung "$
```

(%i25)

```
"+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++"$
```

(%i26)

Berechnung der Kreisfläche




```
(%i1)
"*************************************************************"$
(%i2)
" Bestimmung der Kreisfläche                                  "$
(%i3)
"*************************************************************"$
(%i4)  f(x):=sqrt(r^2-x^2);
(%o4)   f(x) := sqrt(r^2 - x^2)
(%i6)  Flaeche_des_Kreises:4*integrate(f(x),x,0,r);
Is   r   positive, negative, or zero? positive;
(%o6)   %pi r^2
(%i8)  Flaeche_des_Kreises;
(%o8)   %pi r^2
(%i9)
```

```
(%i1)  "*****************************************************"$
(%i2)  " Natürlicher Logarithmus als Integralfunktion       "$
(%i3)  "*****************************************************"$
(%i4)  f(u):=integrate(1/x,x,1,u);
```

$$(\%o4)\quad f(u) := \text{INTEGRATE}\left(\frac{1}{x}, x, 1, u\right)$$

```
(%i5)  f(x);
Is   x - 1   positive, negative, or zero? positive;
```

$$(\%o5)\quad \log(x)$$

```
(%i6)  "*****************************************************"$
(%i7)  f(x);
Is   x - 1   positive, negative, or zero? negative;
Is   x   positive, negative, or zero? negative;
Principal Value
```

$$(\%o7)\quad \log\left(-\frac{1}{x}\right)$$

```
(%i9)  f(x);
Is   x - 1   positive, negative, or zero? negative;
Is   x   positive, negative, or zero? positive;
```

$$(\%o9)\quad \log(x)$$

```
(%i10)  f(x);
Is   x - 1   positive, negative, or zero? negative;
Is   x   positive, negative, or zero? zero;
Principal Value
```

$$(\%o10)\quad \log\left(-\frac{1}{x}\right)$$

```
(%i11)  "und weitere Fallunterscheidungen ...                "$
(%i12)
```

```
(%i1)  "*****************************************************"$
(%i2)  " Partialbruchzerlegung                                "$
(%i3)  "*****************************************************"$
(%i4)  f(x):=(x^4-2*x^2+2)/(x^2+1);
```

$$(\%o4)\quad f(x) := \frac{x^4 - 2\,x^2 + 2}{x^2 + 1}$$

```
(%i5)  integrate(f(x),x);
```

$$(\%o5)\quad 5\,\mathrm{atan}(x) + \frac{x^3 - 9\,x}{3}$$

```
(%i6)  "*****************************************************"$
(%i7)  f(x):=1/(x^2-1);
```

$$(\%o7)\quad f(x) := \frac{1}{x^2 - 1}$$

```
(%i8)  integrate(f(x),x);
```

$$(\%o8)\quad \frac{\log(x - 1)}{2} - \frac{\log(x + 1)}{2}$$

```
(%i9)  "*****************************************************"$
(%i10)  f(x):=1/((x-1)*(x-2)*(x-3));
```

$$(\%o10)\quad f(x) := \frac{1}{(x - 1)(x - 2)(x - 3)}$$

```
(%i11)  integrate(f(x),x);
```

$$(\%o11)\quad \frac{\log(x - 1)}{2} - \log(x - 2) + \frac{\log(x - 3)}{2}$$

```
(%i12)  "*****************************************************"$
(%i13)  f(x):=(x-4)/((x-1)^2*(x+1));
```

$$(\%o13)\quad f(x) := \frac{x - 4}{(x - 1)^2\,(x + 1)}$$

```
(%i14)  integrate(f(x),x);
```

$$(\%o14)\quad -\frac{5\,\log(x + 1)}{4} + \frac{5\,\log(x - 1)}{4} + \frac{3}{2\,x - 2}$$

```
(%i15)  "*****************************************************"$
(%i16)  f(x):=(x+2)/((x^2+4)*(x-1));
```

$$(\%o16)\quad f(x) := \frac{x + 2}{(x^2 + 4)(x - 1)}$$

```
(%i17)  integrate(f(x),x);
```

$$(\%o17)\quad -\frac{3\,\log(x^2 + 4)}{10} + \frac{\mathrm{atan}\left(\frac{x}{2}\right)}{5} + \frac{3\,\log(x - 1)}{5}$$

```
(%i18)  "*****************************************************"$
(%i19)
```

```
(%i1)  "*****************************************************"$
(%i2)  " Partielle Integration                                "$
(%i3)  "*****************************************************"$
(%i4)  f(x):=x*%e^x;
(%o4)  f(x) := x %e^x
(%i5)  integrate(f(x),x);
(%o5)  (x - 1) %e^x
(%i6)  "*****************************************************"$
(%i7)  f(x):=x^2*%e^x;
(%o7)  f(x) := x^2 %e^x
(%i8)  integrate(f(x),x);
(%o8)  (x^2 - 2 x + 2) %e^x
(%i9)  "*****************************************************"$
(%i10)  f(x):=log(x);
(%o10)  f(x) := log(x)
(%i11)  integrate(f(x),x);
(%o11)  x log(x) - x
(%i12)  "*****************************************************"$
(%i13)  f(x):=%e^x*sin(x);
(%o13)  f(x) := %e^x sin(x)
(%i14)  integrate(f(x),x);
                          %e^x (sin(x) - cos(x))
(%o14)                    ----------------------
                                    2
(%i15)  "*****************************************************"$
(%i16)  f(x):=%e^x*cos(x);
(%o16)  f(x) := %e^x cos(x)
(%i17)  integrate(f(x),x);
                          %e^x (sin(x) + cos(x))
(%o17)                    ----------------------
                                    2
(%i18)  "*****************************************************"$
(%i19)  f(x):=cos(x)*sin(x);
(%o19)  f(x) := cos(x) sin(x)
(%i20)  integrate(f(x),x);
                                  cos(x)^2
(%o20)                          - --------
                                    2
(%i21)  "*****************************************************"$
(%i22)  f(x):=(sin(x))^2;
```

(%o22) $f(x) := \sin(x)^2$

(%i23) `integrate(f(x),x);`

(%o23) $$\frac{x - \frac{\sin(2\,x)}{2}}{2}$$

(%i24) `"****************************************************"$`

(%i25) `f(x):=(cos(x))^2;`

(%o25) $f(x) := \cos(x)^2$

(%i26) `integrate(f(x),x);`

(%o26) $$\frac{\frac{\sin(2\,x)}{2} + x}{2}$$

(%i27) `"****************************************************"$`

(%i28) `f(x):=(sin(x))^2+(cos(x))^2;`

(%o28) $f(x) := \sin(x)^2 + \cos(x)^2$

(%i29) `integrate(f(x),x);`

(%o29) $$\frac{\frac{\sin(2\,x)}{2} + x}{2} + \frac{x - \frac{\sin(2\,x)}{2}}{2}$$

(%i32) `trigsimp(%);` Trigonometrischen Ausdruck vereinfachen

(%o32) x

(%i33) `"****************************************************"$`

(%i34)

die gegebene Funktion ist f(x)=1

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(%i1)  "*****************************************************"$
(%i2)  " integralbeispiele "$
(%i3)  "*****************************************************"$
(%i4)  f(x):=(sin(x))^2;
```

$$(\%o4)\quad f(x) := \sin(x)^2$$

```
(%i5)  integrate(f(x),x);
```

$$(\%o5)\quad \frac{x - \frac{\sin(2\,x)}{2}}{2}$$

```
(%i6)  "*****************************************************"$
(%i7)  f(x):=(a^2-x^2)^(1/2);
```

$$(\%o7)\quad f(x) := \left(a^2 - x^2\right)^{1/2}$$

```
(%i8)  integrate(f(x),x);
```

$$(\%o8)\quad \frac{a^2\,\mathrm{asin}\left(\frac{x}{|a|}\right)}{2} + \frac{x\sqrt{a^2 - x^2}}{2}$$

```
(%i9)  "*****************************************************"$
(%i10)  f(x):=(a^2-x^2)^(-1/2);
```

$$(\%o10)\quad f(x) := \left(a^2 - x^2\right)^{\frac{-1}{2}}$$

```
(%i11)  integrate(f(x),x);
```

$$(\%o11)\quad \mathrm{asin}\left(\frac{x}{|a|}\right)$$

```
(%i12)  "*****************************************************"$
(%i13)
```

```
(%i1)  "*****************************************************"$
(%i2)  " Zerlegung einer quadratischen Funktion              "$
(%i3)  "*****************************************************"$
(%i4)  f(x):=x^2-8*x+15;
```

$$(\%o4)\quad f(x) := x^2 - 8\,x + 15$$

```
(%i5)  g(x):=x^2;
```

$$(\%o5)\quad g(x) := x^2$$

```
(%i6)  h(x):=8*x-15;
```

$$(\%o6)\quad h(x) := 8\,x - 15$$

```
(%i7)  g:g(x)=h(x);
```

$$(\%o7)\quad x^2 = 8\,x - 15$$

```
(%i8)  "*****************************************************"$
(%i9)  " Das ist mittels Iterationsverfahren verarbeitbar    "$
(%i10)  "*****************************************************"$
(%i11)  x[0]:10;
(%o11)   10
(%i12)  f(n):=x[n+1]:(8*x[n]-15)/x[n];
```

$$(\%o12)\quad f(n) := x_{n+1} : \frac{8\,x_n - 15}{x_n}$$

Iterationsverfahren

```
(%i13)  f(0);
```

$$(\%o13)\quad \frac{13}{2}$$

```
(%i14)  f(0),numer;
(%o14)   6.5
(%i15)  f(1);
(%o15)   5.6923076923076925
(%i16)  f(2);
(%o16)   5.3648648648648649
(%i17)  f(3);
(%o17)   5.2040302267002518
(%i18)  f(4);
(%o18)   5.1176185866408517
(%i19)  f(5);
(%o19)   5.06894921025253
(%i20)  f(6);
(%o20)   5.040806806732097
(%i21)  "*****************************************************"$
(%i22)  " Erste Lösung x=5                                   "$
```

```
(%i23)  "*****************************************************"$
(%i24)  x[0]:-10;
(%o24)   - 10
(%i25)  f(0);
(%o25)  19/2
(%i26)  f(0),numer;
(%o26)   9.5
(%i27)  f(1);
(%o27)   6.4210526315789478
(%i28)  f(2);
(%o28)   5.6639344262295079
(%i29)  f(3);
(%o29)   5.3516642547033282
(%i30)  f(4);
(%o30)   5.1971335857220113
(%i31)  f(5);
(%o31)   5.1137936417087255
(%i32)  f(6);
(%o32)   5.0667568832543086
(%i33)  f(7);
(%o33)   5.0395263981235852
(%i34)  "*****************************************************"$
(%i35)  " Wieder bekommen wir 5                                "$
(%i36)  "*****************************************************"$
(%i37)  x[0]:1;
(%o37)   1
(%i38)  f(0);
(%o38)   - 7
(%i39)  f(1);
(%o39)  71/7
(%i40)  "*****************************************************"$
(%i41)  x[0]:0;
(%o41)   0
(%i42)  f(0);
Division by 0
#0: f(n=0)
 -- an error.  Quitting.  To debug this try DEBUGMODE(TRUE);
```

```
(%i43)  x[0]:2;
(%o43)   2
(%i44)  f(0);
(%o44)   1/2
(%i45)  f(1);
(%o45)   - 22
(%i46)  f(2);
(%o46)   191/22
(%i47)  f(2),numer;
(%o47)   8.6818181818181817
(%i48)  "*****************************************************"$
(%i49)  x[0]:4;
(%o49)   4
(%i50)  f(0);
(%o50)   17/4
(%i51)  "*****************************************************"$
(%i52)  x[0]:3.5;
(%o52)   3.5
(%i53)  f(0);
(%o53)   3.7142857142857144
(%i54)  f(1);
(%o54)   3.9615384615384617
(%i55)  f(2);
(%o55)   4.2135922330097086
(%i56)  "*****************************************************"$
(%i57)  x[0]:3.1;
(%o57)   3.1000000000000001
(%i58)  f(0);
(%o58)   3.1612903225806455
(%i59)  f(1);
(%o59)   3.2551020408163271
(%i60)  f(2);
(%o60)   3.3918495297805653
(%i61)  "*****************************************************"$
(%i62)  x[0]:2.8;
```

```
(%o62)   2.7999999999999998
(%i63)  f(0);
(%o63)   2.6428571428571423
(%i64)  f(1);
(%o64)   2.3243243243243232
(%i65)  f(2);
(%o65)   1.5465116279069737
(%i66)  f(3);
(%o66)   - 1.6992481203007708
(%i67)  f(4);
(%o67)   16.827433628318488
(%i68)  f(5);
(%o68)   7.1085984748882414
(%i69)  "*****************************************************"$
(%i70)  " Es ist offenbar mit diesem Iterationsverfahren     "$
(%i74)  " nicht möglich, die zweite Nullstelle zu            "$
(%i75)  " erreichen                                          "$
(%i76)  "*****************************************************"$
(%i77)
```

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(%i1)  "*****************************************************"$
(%i2)  " Eindeutige Lösbarkeit Gleichungssystem              "$
(%i3)  "*****************************************************"$
(%i4)  g1:x+y+z=3;
(%o4)   z + y + x = 3
(%i5)  g2:2*x-3*y+4*z=3;
(%o5)   4 z - 3 y + 2 x = 3
(%i6)  g3:8*x-2*y+z=7;
(%o6)   z - 2 y + 8 x = 7
(%i7)  "*****************************************************"$
(%i8)  A:coefmatrix([g1,g2,g3],[x,y,z]);
                                [ 1   1   1 ]
(%o8)                           [           ]
                                [ 2  - 3  4 ]
                                [           ]
                                [ 8  - 2  1 ]
(%i9)  b:matrix([3], [3], [7]);
                                [ 3 ]
                                [   ]
(%o9)                           [ 3 ]
                                [   ]
                                [ 7 ]
(%i10)  C:invert(A);
                                [  1     3     7  ]
                                [ ---   - --   --  ]
                                [ 11     55    55 ]
                                [                 ]
                                [  6     7     2  ]
(%o10)                          [ ---   - --  - --  ]
                                [ 11     55    55 ]
                                [                 ]
                                [  4     2     1  ]
                                [ ---    --  - --  ]
                                [ 11     11    11 ]
(%i11)  d:C.b;
                                [ 1 ]
                                [   ]
(%o11)                          [ 1 ]
                                [   ]
                                [ 1 ]
(%i12)  "*****************************************************"$
(%i13)  " Das ist der Lösungsvektor                          "$
(%i14)  "*****************************************************"$
```

```
(%i15)  determinant(A);
(%o15)   55
(%i16)  "*************************************************"$
(%i17)  " Wenn die Koeffizientendeterminante von NULL       "$
(%i18)  " verschieden ist, ist das Gleichungssystem eindeutig"$
(%i19)  " lösbar                                            "$
(%i20)  "*************************************************"$
(%i21)
```

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(%i1)  "*****************************************************"$
(%i2)  " Binomialkoeffizient benutzerdefiniert "$
(%i4)  "*****************************************************"$
(%i5)  c(n,k):=n!/(k!*(n-k)!);
(%o5)   c(n , k) := n !
                    ---------
                    k ! (n - k) !
(%i6)  "*****************************************************"$
(%i7)  " Anwendungen: "$
(%i8)  "*****************************************************"$
(%i9)  c(45,6);
(%o9)   8145060
(%i10)  "*****************************************************"$
(%i11)  " Möglichkeiten österreichisches Lotto "$
(%i12)  "*****************************************************"$
(%i13)  c(49,6);
(%o13)   13983816
(%i14)  "*****************************************************"$
(%i15)  " Möglichkeiten deutsches Lotto "$
(%i16)  "*****************************************************"$
(%i17)  c(50,5);
(%o17)   2118760
(%i18)  "*****************************************************"$
(%i19)  " Das hat was mit dem europäischen Lotto zu tun "$
(%i20)  "*****************************************************"$
(%i21)
```

Sechs aus 49, deutsches Lotto

```
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(%i1)  "*****************************************************"$
(%i2)  " Identität Binomialkoeffizienten "$
(%i3)  "*****************************************************"$
(%i4)  f(n,k):=binomial(n,k)=n!/(k!*(n-k)!);
```

(%o4) $$f(n , k) := \binom{n}{k} = \frac{n !}{k ! (n - k) !}$$

```
(%i5)  n:0;
(%o5)   0
(%i6)  makelist(f(n,k),k,0,n);
(%o6)   [ 1 = 1 ]
(%i7)  n:1;
(%o7)   1
(%i8)  makelist(f(n,k),k,0,n);
(%o8)   [ 1 = 1 , 1 = 1 ]
(%i9)  n:2;
(%o9)   2
(%i10)  makelist(f(n,k),k,0,n);
(%o10)   [ 1 = 1 , 2 = 2 , 1 = 1 ]
(%i11)  n:3;
(%o11)   3
(%i12)  makelist(f(n,k),k,0,n);
(%o12)   [ 1 = 1 , 3 = 3 , 3 = 3 , 1 = 1 ]
(%i13)  n:4;
(%o13)   4
(%i14)  makelist(f(n,k),k,0,n);
(%o14)   [ 1 = 1 , 4 = 4 , 6 = 6 , 4 = 4 , 1 = 1 ]
(%i15)  n:5;
(%o15)   5
(%i16)  makelist(f(n,k),k,0,n);
(%o16)   [ 1 = 1 , 5 = 5 , 10 = 10 , 10 = 10 , 5 = 5 , 1 = 1 ]
(%i17)  n:6;
(%o17)   6
(%i18)  makelist(f(n,k),k,0,n);
```

```
(%o18)   [ 1 = 1 , 6 = 6 , 15 = 15 , 20 = 20 , 15 = 15 , 6 = 6 , 1 = 1 ]
(%i19)  n:7;
(%o19)   7
(%i20)  makelist(f(n,k),k,0,n);
(%o20)   [ 1 = 1 , 7 = 7 , 21 = 21 , 35 = 35 , 35 = 35 , 21 = 21 , 7 = 7 , 1 = 1 ]
(%i21)
```

```
(%i1)  "*****************************************************"$
(%i2)  " Zeilensummen Pascal-Dreieck                         "$
(%i3)  "*****************************************************"$
(%i6)  c(n,k):=n!/(k!*(n-k)!);
```

$$(\%o6)\quad c(n,k) := \frac{n!}{k!\,(n-k)!}$$

```
(%i7)  f(n):=sum(c(n,i),i,0,n);
```

$$(\%o7)\quad f(n) := \mathrm{SUM}(c(n,i),i,0,n)$$

```
(%i8)  "*****************************************************"$
(%i9)  f(1);
(%o9)   2
(%i10)  f(2);
(%o10)   4
(%i11)  f(3);
(%o11)   8
(%i12)  f(4);
(%o12)   16
(%i13)  f(5);
(%o13)   32
(%i14)  f(6);
(%o14)   64
(%i15)
```

```
(%i1)  "*****************************************************"$
(%i2)  " Mächtigkeit Potenzmenge                             "$
(%i3)  "*****************************************************"$
(%i4)  sum(binomial(n,k),k,0,n);
```

$$(\%o4)\quad \sum_{k=0}^{n} \binom{n}{k}$$

```
(%i5)  %,simpsum;
```

$$(\%o5)\quad 2^{n}$$

```
(%i6)  "*****************************************************"$
(%i7)  " Die Potenzmenge ist die Menge aller Teilmengen      "$
(%i8)  " einer gegebenen Menge. Unter Mächtigkeit            "$
(%i9)  " versteht man die Anzahl ihrer Elemente              "$
(%i10)  "*****************************************************"$
(%i11)
```

```
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```

```
(%i1)  "*****************************************************"$
(%i2)  " Zeilensummen im Pascalschen Dreieck "$
(%i3)  "*****************************************************"$
(%i4)  sum(binom(n,k),k,0,n);
```

$$(\%o4)\quad \sum_{k=0}^{n} \binom{n}{k}$$

```
(%i5)  %,simpsum;
```

$$(\%o5)\quad 2^{n}$$

```
(%i6)  "*****************************************************"$
(%i7)  sum(binomial(n,k),k,0,n);
```

$$(\%o7)\quad \sum_{k=0}^{n} \binom{n}{k}$$

```
(%i8)  %,simpsum;
```

$$(\%o8)\quad 2^{n}$$

```
(%i9)  "*****************************************************"$
(%i10)  c(n,k):=n!/(k!*(n-k)!);
```

$$(\%o10)\quad c\left(n\ ,\ k\right) := \frac{n\ !}{k\ !\ \left(n\ -\ k\right)\ !}$$

```
(%i11)  sum(c(n,k),k,0,n);
```

$$(\%o11)\quad \sum_{k=0}^{n} c\left(n\ ,\ k\right)$$

```
(%i12)  %,simpsum;
```

$$(\%o12)\quad \sum_{k=0}^{n} c\left(n\ ,\ k\right)$$

```
(%i13)  "*****************************************************"$
(%i14)  " Diese Summe kann NICHT ausgewertet werden "$
```

```
(%i1)  "*************************************************"$
(%i2)  " Betriebsoptimum als Lösung der Gleichung       "$
(%i3)  " Grenzkosten = Durchschnittskosten              "$
(%i4)  "*************************************************"$
(%i5)  K(x):=x^2+8*x+36;
```

$$(\%o5)\quad K(x) := x^2 + 8\,x + 36$$

```
(%i7)  DK(x):=K(x)/x;
```

$$(\%o7)\quad DK(x) := \frac{K(x)}{x}$$

```
(%i9)  GK(x):=diff(K(x),x);
```

$$(\%o9)\quad GK(x) := DIFF(K(x),\,x)$$

```
(%i10)  solve(GK(x)=DK(x),x);
```

$$(\%o10)\quad [\,x = -6\,,\ x = 6\,]$$

```
(%i11)  "*************************************************"$
(%i12)  " Das Betriebsoptimum ist 6                     "$
(%i13)  "*************************************************"$
(%i14)  K(x):=x^2+2*x+64;
```

$$(\%o14)\quad K(x) := x^2 + 2\,x + 64$$

```
(%i15)  GK(x):=diff(K(x),x);
```

$$(\%o15)\quad GK(x) := DIFF(K(x),\,x)$$

```
(%i16)  DK(x):=K(x)/x;
```

$$(\%o16)\quad DK(x) := \frac{K(x)}{x}$$

```
(%i17)  solve(GK(x)=DK(x),x);
```

$$(\%o17)\quad [\,x = -8\,,\ x = 8\,]$$

```
(%i18)  "*************************************************"$
(%i19)  " Das Betriebsoptimum ist 8                     "$
(%i20)  "*************************************************"$
(%i23)  kill(all);
(%o0)   DONE
(%i1)  DK(x):=K(x)/x;
```

$$(\%o1)\quad DK(x) := \frac{K(x)}{x}$$

```
(%i2)  ab:diff(DK(x),x);
```

$$(\%o2)\quad \frac{\frac{d}{d\,x}K(x)}{x} - \frac{K(x)}{x^2}$$

```
(%i3)  solve(ab=0,x);
```

(%o3) $$[\ x = \frac{K(x)}{\frac{d}{d\ x} K(x)}\ ]$$

```
(%i4)  g:%;
```

(%o4) $$[\ x = \frac{K(x)}{\frac{d}{d\ x} K(x)}\ ]$$

```
(%i5)  g:g/K(x);
```

(%o5) $$[\ \frac{x}{K(x)} = \frac{1}{\frac{d}{d\ x} K(x)}\ ]$$

```
(%i6)  g:1/g;
```

(%o6) $$[\ \frac{K(x)}{x} = \frac{d}{d\ x} K(x)\ ]$$

```
(%i7)  "=====================================================$
(%i8)  " Durchschnittskosten = Grenzkosten                   "$
(%i9)  "=====================================================$
(%i10)
```

```
(%i1)  "#########################################################"$
(%i2)  "# Grenzkosten gegeben                                   #"$
(%i3)  "#########################################################"$
(%i4)  F:10000;
(%o4)   10000
(%i5)  GK(x):=2*x+8;
(%o5)   GK(x) := 2 x + 8
(%i6)  KV:integrate(GK(x),x);
(%o6)   x^2 + 8 x
(%i7)  K(x):=KV+F;
(%o7)   K(x) := KV + F
(%i8)  K(x);
(%o8)   x^2 + 8 x + 10000
(%i9)  "#########################################################"$
(%i10)  F:20000;
(%o10)   20000
(%i11)  KV:1.2*x+8;
(%o11)   1.2 x + 8
(%i12)  KV:integrate(GK(x),x);
(%o12)   x^2 + 8 x
(%i13)  K(x):=KV+F;
(%o13)   K(x) := KV + F
(%i14)  K(x);
(%o14)   x^2 + 8 x + 20000
(%i15)  "#########################################################"$
(%i16)  F:55000;
(%o16)   55000
(%i17)  GK(x):=0.2*x^2-1.2*x+15;
(%o17)   GK(x) := 0.20000000000000001 x^2 - 1.2 x + 15
(%i18)  KV:integrate(GK(x),x);
(%o18)   0.066666666666666666 x^3 - 0.59999999999999998 x^2 + 15 x
(%i19)  K(x):=KV+F;
(%o19)   K(x) := KV + F
(%i20)  K(x);
(%o20)   0.066666666666666666 x^3 - 0.59999999999999998 x^2 + 15 x + 55000
(%i21)  "#########################################################"$
(%i22)
```

```
(%i1)  "*****************************************************"$
(%i2)  " Grenzkosten gegeben - Betriebsoptimum gesucht       "$
(%i3)  "*****************************************************"$
(%i4)  F:10000;
(%o4)   10000
(%i5)  GK(x):=2*x+8;
(%o5)   GK(x) := 2 x + 8
(%i6)  KV:integrate(GK(x),x);
(%o6)   x^2 + 8 x
(%i7)  K(x):=KV+F;
(%o7)   K(x) := KV + F
(%i8)  K(x);
(%o8)   x^2 + 8 x + 10000
(%i9)  "*****************************************************"$
(%i10)  " Das sind die Gesamtkosten                           "$
(%i11)  "*****************************************************"$
(%i12)  DK(x):=K(x)/x;
(%o12)   DK(x) := K(x)/x
(%i13)  ab:diff(DK(x),x);
(%o13)   (2 x + 8)/x - (x^2 + 8 x + 10000)/x^2
(%i14)  solve(ab=0,x);
(%o14)   [ x = - 100 , x = 100 ]
(%i15)  "*****************************************************"$
(%i16)  " Das Betriebsoptimum ist 100                         "$
(%i18)  "*****************************************************"$
(%i20)
```

```
(%i1)  "*****************************************************"$
(%i2)  " Nachfrage ist Funktion des Preises                  "$
(%i3)  "*****************************************************"$
(%i4)  x(p):=16-p^2;
```

$$(\%o4)\quad x(p) := 16 - p^2$$

```
(%i5)  U(p):=p*x(p);
```

$$(\%o5)\quad U(p) := p\,x(p)$$

```
(%i6)  ab:diff(U(p),p);
```

$$(\%o6)\quad 16 - 3\,p^2$$

```
(%i7)  solve(ab=0,p);
```

$$(\%o7)\quad \left[p = -\frac{4}{\sqrt{3}},\ p = \frac{4}{\sqrt{3}}\right]$$

```
(%i8)  "*****************************************************"$
(%i9)  " Der positive Wert ist der umsatzmaximale Preis      "$
(%i10)  "*****************************************************"$
(%i11)  diff(U(p),p,2);
```

$$(\%o11)\quad -6\,p$$

```
(%i12)  %,p=4/sqrt(3);
```

$$(\%o12)\quad -\frac{24}{\sqrt{3}}$$

```
(%i13)  "*****************************************************"$
(%i14)  " Wenn die zweite Ableitung negativ ist, liegt ein   "$
(%i15)  " Maximum vor                                        "$
(%i16)  "*****************************************************"$
(%i17)  U(4/sqrt(3));
```

$$(\%o17)\quad \frac{128}{3\sqrt{3}}$$

```
(%i18)  "*****************************************************"$
(%i19)  " Das ist der maximale Umsatz                        "$
(%i20)  "*****************************************************"$
(%i21)  x(4/sqrt(3));
```

$$(\%o21)\quad \frac{32}{3}$$

```
(%i22)  "*****************************************************"$
(%i23)  " Das ist die optimale Nachfrage                     "$
(%i24)  "*****************************************************"$
(%i25)
```

```
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```

(%i1)
```
"*****************************************************"$
```

(%i3)
```
" Kurvendiskussion                                    "$
```

(%i4)
```
"*****************************************************"$
```

(%i5)
```
f(x):=(1-x)/sqrt(2*x-x^2);
```

(%o5)
$$f(x) := \frac{1 - x}{\sqrt{2\,x - x^2}}$$

(%i6)
```
"*****************************************************"$
```

(%i7)
```
solve(f(x),x);
```

(%o7)
$$[\,x = 1\,]$$

(%i8)
```
ab:diff(f(x),x);
```

(%o8)
$$-\frac{1}{\sqrt{2\,x - x^2}} - \frac{(2 - 2\,x)(1 - x)}{2\,(2\,x - x^2)^{3/2}}$$

(%i9)
```
solve(ab=0,x);
```

(%o9)
$$[\,]$$

(%i10)
```
ab2:diff(f(x),x,2);
```

(%o10)
$$\frac{1 - x}{(2\,x - x^2)^{3/2}} + \frac{2 - 2\,x}{(2\,x - x^2)^{3/2}} + \frac{3\,(2 - 2\,x)^2\,(1 - x)}{4\,(2\,x - x^2)^{5/2}}$$

(%i11)
```
solve(ab2=0,x);
```

(%o11)
$$[\,x = 1\,]$$

(%i12)
```
nenner:denom(f(x));
```

(%o12)
$$\sqrt{2\,x - x^2}$$

(%i13)
```
solve(nenner=0,x);
```

(%o13)
$$[\,x = 0\,,\ x = 2\,]$$

(%i14)

```
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(%i1)  "*****************************************************"$
(%i2)  " Nicht immer gibt es Pole                             "$
(%i3)  "*****************************************************"$
(%i4)  f(x):=(x^3-6*x^2+12*x-8)/(x^2-4*x+4);
```

$$(\%o4)\quad f(x) := \frac{x^3 - 6\,x^2 + 12\,x - 8}{x^2 - 4\,x + 4}$$

```
(%i5)  zaehler:num(f(x),x);
```

$$(\%o5)\quad x^3 - 6\,x^2 + 12\,x - 8$$

```
(%i6)  nenner:denom(f(x),x);
```

$$(\%o6)\quad x^2 - 4\,x + 4$$

```
(%i7)  "*****************************************************"$
(%i8)  solve(zaehler=0,x);
```

$$(\%o8)\quad [\,x = 2\,]$$

```
(%i9)  solve(nenner=0,x);
```

$$(\%o9)\quad [\,x = 2\,]$$

```
(%i10)  "*****************************************************"$
(%i11)  factor(f(x));
```

$$(\%o11)\quad x - 2$$

```
(%i12)  "*****************************************************"$
(%i13)  " Es gibt nur eine Nullstelle, keinen Pol             "$
(%i14)  "*****************************************************"$
(%i15)
```

```
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provides bug reporting information.
(%i1)  "*****************************************************"$
(%i2)  " Nullstellen eines Polynoms "$
(%i3)  "*****************************************************"$
(%i4)  p3(x):=x^3+x-1;
(%o4)  p3(x) := x^3 + x - 1
(%i5)  "*****************************************************"$
(%i6)  allroots(p3(x));
(%o6)  [ x = 0.68232780382801939 , x = 1.1615413999972519 %i -
0.34116390191400969 , x = - 1.1615413999972519 %i - 0.34116390191400969 ]
(%i7)  "*****************************************************"$
(%i8)  " Das ist die Lösung mit ALLROOTS "$
(%i9)  "*****************************************************"$
(%i10)  solve(p3(x)=0,x);
```

$$(\%o10)\quad \left[x = \left(\frac{\sqrt{31}}{6\sqrt{3}} + \frac{1}{2}\right)^{1/3}\left(-\frac{\sqrt{3}\,\%i}{2} - \frac{1}{2}\right) - \frac{\frac{\sqrt{3}\,\%i}{2} - \frac{1}{2}}{3\left(\frac{\sqrt{31}}{6\sqrt{3}} + \frac{1}{2}\right)^{1/3}},\ x = \left(\frac{\sqrt{31}}{6\sqrt{3}} + \frac{1}{2}\right)^{1/3}\left(\frac{\sqrt{3}\,\%i}{2} - \frac{1}{2}\right) - \frac{-\frac{\sqrt{3}\,\%i}{2} - \frac{1}{2}}{3\left(\frac{\sqrt{31}}{6\sqrt{3}} + \frac{1}{2}\right)^{1/3}},\ x = \left(\frac{\sqrt{31}}{6\sqrt{3}} + \frac{1}{2}\right)^{1/3} - \frac{1}{3\left(\frac{\sqrt{31}}{6\sqrt{3}} + \frac{1}{2}\right)^{1/3}} \right]$$

```
(%i11)  %,numer;
(%o11)  [ x = 1.0117801418773178 (- 0.8660254037844386 %i - 0.5) -
0.32945233804929852 (0.8660254037844386 %i - 0.5), x = 1.0117801418773178
(0.8660254037844386 %i - 0.5) - 0.32945233804929852
(- 0.8660254037844386 %i - 0.5), x = 0.68232780382801927 ]
(%i12)  "*****************************************************"$
(%i13)  " Das ist die Lösung mit SOLVE "$
(%i14)  "*****************************************************"$
(%i15)
```

```
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```

(%i1)
```
"**************************************************"$
```

(%i2)
```
" Nullstellen und Pole "$
```

(%i3)
```
"**************************************************"$
```

(%i4)
```
y(x):=(x^4-3*x^3+3*x^2-x)/(x^3+x^2-x-1);
```

(%o4)
$$y(x) := \frac{x^4 - 3\,x^3 + 3\,x^2 - x}{x^3 + x^2 - x - 1}$$

wo der Zähler NULL ist, gibt es wahrscheinlich Nullstellen, wo der Nenner NULL ist, gibt es wahrscheinlich Pole

(%i5)
```
zaehler:num(y(x));
```

(%o5)
$$x^4 - 3\,x^3 + 3\,x^2 - x$$

(%i6)
```
nenner:denom(y(x));
```

(%o6)
$$x^3 + x^2 - x - 1$$

(%i7)
```
"**************************************************"$
```

(%i8)
```
" Zerlegung in Zaehler und Nenner "$
```

(%i9)
```
"**************************************************"$
```

(%i10)
```
solve(zaehler=0,x);
```

(%o10)
$$[\,x = 1\,,\,x = 0\,]$$

(%i12)
```
solve(nenner=0,x);
```

(%o12)
$$[\,x = 1\,,\,x = -1\,]$$

(%i13)
```
factor(zaehler);
```

(%o13)
$$(x - 1)^3\,x$$

(%i14)
```
factor(nenner);
```

(%o14)
$$(x - 1)\,(x + 1)^2$$

(%i15)
```
"**************************************************"$
```

(%i16)
```
" Man kann also durch x-1 kürzen, x=1 ist kein Pol "$
```

(%i17)
```
"**************************************************"$
```

(%i19)
```
factor(y(x));
```

(%o19)
$$\frac{(x - 1)^2\,x}{(x + 1)^2}$$

(%i20)
```
f(x):=(x-1)^2*x/(x+1)^2;
```

(%o20)
$$f(x) := \frac{(x - 1)^2\,x}{(x + 1)^2}$$

(%i21)
```
f(1);
```

(%o21)
$$0$$

```
(%i22)  f(0);
(%o22)   0
(%i23)  "*****************************************************"$
(%i24)  " Das muss ja sein, weil es Nullstellen sind "$
(%i25)  "*****************************************************"$
(%i26)
```

```
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(%i1)  "*****************************************************"$
(%i2)  " Nullstellen und Pole "$
(%i3)  "*****************************************************"$
(%i4)  f(x):=x^3/(x^3-2*x^2-4*x+8);
                                  3
(%o4)  f(x) := ------------------------------
                 3      2
               x  - 2 x  + (- 4) x + 8
(%i5)  zaehler:num(f(x));
                3
(%o5)          x
(%i6)  nenner:denom(f(x));
                3      2
(%o6)          x  - 2 x  - 4 x + 8
(%i7)  "*****************************************************"$
(%i8)  " Zerlegung in Zähler und Nenner "$
(%i9)  "*****************************************************"$
(%i10)  solve(zaehler=0,x);
(%o10)   [ x = 0 ]
(%i11)  solve(nenner=0,x);
(%o11)   [ x = - 2 , x = 2 ]
(%i12)  f(0);
(%o12)   0
(%i14)  "*****************************************************"$
(%i15)  " Am Punkt N(0(0) ist eine Nullstelle "$
(%i16)  " Es gibt zwei Pole x=-2 und x=2 "$
(%i17)  "*****************************************************"$
(%i18)
```

```
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Dedicated to the memory of William Schelter.
This is a development version of Maxima. The function bug_report()
provides bug reporting information.
(%i1)  "*****************************************************"$
(%i2)  " Nullstellen und Pole "$
(%i3)  "*****************************************************"$
(%i4)  f(x):=10*x^2/(x^4+5*x^3+7*x^2+5*x+6);
                                  10 x^2
(%o4)  f(x) := ---------------------------------
               x^4 + 5 x^3 + 7 x^2 + 5 x + 6
(%i5)  zaehler:num(f(x));
                                  2
(%o5)  10 x
(%i6)  nenner:denom(f(x));
                    4      3      2
(%o6)  x^4 + 5 x^3 + 7 x^2 + 5 x + 6
(%i7)  solve(zaehler=0,x);
(%o7)  [ x = 0 ]
(%i8)  solve(nenner=0,x);
(%o8)  [ x = - 3 , x = - 2 , x = - %i , x = %i ]
(%i9)  "*****************************************************"$
(%i10)  " Es gibt eine Nullstelle und zwei Pole "$
(%i11)  "*****************************************************"$
(%i12)
```

```
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(%i1)  "*************************************************"$
(%i2)  " Nullstellen und Pole "$
(%i3)  "*************************************************"$
(%i4)  f(x):=(x^4-4*x^3+12*x^2+4*x-13)/(2*x^3-4*x^2-8*x+16);
```

$$(\%o4)\quad f(x) := \frac{x^4 - 4\,x^3 + 12\,x^2 + 4\,x - 13}{2\,x^3 - 4\,x^2 + (-8)\,x + 16}$$

```
(%i5)  zaehler:num(f(x));
```

$$(\%o5)\quad x^4 - 4\,x^3 + 12\,x^2 + 4\,x - 13$$

```
(%i6)  nenner:denom(f(x));
```

$$(\%o6)\quad 2\,x^3 - 4\,x^2 - 8\,x + 16$$

```
(%i7)  allroots(zaehler);
(%o7)   [ x = 0.9999999999999989 , x = - 1.0 , x = 3.0 %i + 2.0 , x = 2.0 -
3.0 %i ]
(%i8)  allroots(nenner);
(%o8)   [ x = 1.9999999701976905 , x = 2.0000000298023353 , x = -
2.0000000000000258 ]
(%i9)  solve(zaehler=0,x);
(%o9)   [ x = 1 , x = - 1 , x = 2 - 3 %i , x = 3 %i + 2 ]
(%i10)  solve(nenner=0,x);
(%o10)   [ x = - 2 , x = 2 ]
(%i11)  "*************************************************"$
(%i12)  " Das sind "$
(%i13)  " a) die Nullstellen (%o9)"$
(%i14)  " b) die Pole (%o10)"$
(%i15)  "*************************************************"$
(%i16)
```

```
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(%i1)  "*****************************************************"$
(%i2)  " Umgekehrte Kurvendiskussion "$
(%i3)  "*****************************************************"$
(%i4)  f(x):=a*x^4+b*x^3+c*x^2+d*x+e;
(%o4)   f(x) := a x^4 + b x^3 + c x^2 + d x + e
(%i5)  ab:diff(f(x),x);
(%o5)   4 a x^3 + 3 b x^2 + 2 c x + d
(%i6)  ab,x=0;
(%o6)   d
(%i7)  g:%=2;
(%o7)   d = 2
(%i8)  "*****************************************************"$
(%i9)  " Gesucht: Polynom 4. Grades "$
(%i10)  " Für x=0 gibt es eine Tangente mit der Steigung 2 "$
(%i11)  "*****************************************************"$
(%i12)  ab2:diff(f(x),x,2);
(%o12)   12 a x^2 + 6 b x + 2 c
(%i14)  ab2,x=0;
(%o14)   2 c
(%i15)  g:%=-282;
(%o15)   2 c = - 282
(%i16)  solve(g,c);
(%o16)   [ c = - 141 ]
(%i17)  "*****************************************************"$
(%i18)  " Für x=0 ist die zweite Ableitung -282 "$
(%i19)  "*****************************************************"$
(%i20)  ab3:diff(f(x),x,3);
(%o20)   24 a x + 6 b
(%i21)  ab3,x=0;
(%o21)   6 b
(%i22)  g:%=12;
(%o22)   6 b = 12
```

```
(%i23)  solve(g,b);
(%o23)   [ b = 2 ]
(%i24)  "*****************************************************"$
(%i25)  " Für x=0 ist die dritte Ableitung 12 "$
(%i26)  "*****************************************************"$
(%i27)  ab4:diff(f(x),x,4);
(%o27)   24 a
(%i28)  ab4,x=0;
(%o28)   24 a
(%i29)  g:%=24;
(%o29)   24 a = 24
(%i30)  solve(g,a);
(%o30)   [ a = 1 ]
(%i31)  "*****************************************************"$
(%i32)  " Für x=0 ist die vierte Ableitung 24 "$
(%i33)  "*****************************************************"$
(%i34)  f(1)=-135;
(%o34)   e + d + c + b + a = - 135
(%i35)  g1:%;
(%o35)   e + d + c + b + a = - 135
(%i36)  g2:a=1;
(%o36)   a = 1
(%i37)  g3:b=2;
(%o37)   b = 2
(%i38)  g4:c=-141;
(%o38)   c = - 141
(%i39)  g5:d=2;
(%o39)   d = 2
(%i40)  solve([g1,g2,g3,g4,g5],[a,b,c,d,e]);
(%o40)   [ [ a = 1 , b = 2 , c = - 141 , d = 2 , e = 1 ] ]
(%i41)  "*****************************************************"$
(%i42)  " Damit kennen wir auch e = 1 "$
(%i43)  "*****************************************************"$
(%i44)  e:1;
(%o44)   1
(%i47)  f(x):=x^4+2*x^3-141*x^2+2*x+1;
                                4    3          2
(%o47)   f(x) := x  + 2 x  + (- 141) x  + 2 x + 1
(%i48)  "*****************************************************"$
```

```
(%i1)  "Bestimme die optimale Losgröße (Wilsonsche Formel):
x = Losgröße
k0 = Fixkosten
k1 = proportionale Kosten
h = Lagerkosten pro Stück und Zeiteinheit
m = die als konstant angenommene Nachfrage pro Zeiteinheit
"$
```

ein schönes Beispiel aus der Betriebswirtschaftslehre

```
(%i2)  K(x):=k0+k1*x+h/(2*m)*x^2;
```

$$(\%o2)\quad K(x) := k0 + k1\ x + \frac{h}{2\ m}\ x^2$$

```
(%i3)  DK(x):=K(x)/x;
```

$$(\%o3)\quad DK(x) := \frac{K(x)}{x}$$

```
(%i4)  ab:diff(DK(x),x);
```

$$(\%o4)\quad \frac{\frac{h\ x}{m} + k1}{x} - \frac{\frac{h\ x^2}{2\ m} + k1\ x + k0}{x^2}$$

```
(%i5)  solve(ab=0,x);
```

$$(\%o5)\quad [\ x = -\sqrt{2}\ \sqrt{\frac{k0\ m}{h}}\ ,\ x = \sqrt{2}\ \sqrt{\frac{k0\ m}{h}}\ ]$$

```
(%i6)  "**************************************************"$
(%i7)  " Positive Lösung: Wilsonsche Formel                "$
(%i8)  "**************************************************"$
(%i9)
```

```
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(%i1)  Kosten_einer_Bestellung:1000;
(%o1)   1000
(%i2)  Anzahl_der_Bestellungen:n;
(%o2)   n
(%i3)  Bestellkosten:Kosten_einer_Bestellung*Anzahl_der_Bestellungen;
(%o3)   1000 n
(%i4)  "******************************************************"$
(%i5)  Jahresbedarf:25000;
(%o5)   25000
(%i6)  Bestellmenge:Jahresbedarf/Anzahl_der_Bestellungen;
(%o6)   25000/n
(%i7)  "******************************************************"$
(%i8)  durchschnittlicher_Lagerbestand:Bestellmenge/2;
(%o8)   12500/n
(%i9)  "******************************************************"$
(%i10)  Preis_Einheit:30;
(%o10)   30
(%i11)  Zinssatz:10/100;
(%o11)   1/10
(%i12)
durchschnittliche_Kapitalbindung:durchschnittlicher_Lagerbestand*Preis_Einheit
(%o12)   375000/n
(%i13)
Kosten_der_Kapitalbindung:durchschnittliche_Kapitalbindung*Zinssatz;
(%o13)   37500/n
(%i14)  "******************************************************"$
(%i15)  Lagerhaltungskosten:Bestellkosten+Kosten_der_Kapitalbindung;
(%o15)   1000 n + 37500/n
```

(%i16) ab:diff(Lagerhaltungskosten,n);

(%o16) $1000 - \frac{37500}{n^2}$

(%i17) solve(ab=0,n);

(%o17) $[n = - \frac{5\sqrt{3}}{\sqrt{2}} , n = \frac{5\sqrt{3}}{\sqrt{2}}]$

(%i18) %,numer;

(%o18) [n = - 6.1237243569579451 , n = 6.1237243569579451]

(%i20) "**********"$

(%i22) ab2:diff(Lagerhaltungskosten,n,2);

(%o22) $\frac{75000}{n^3}$

(%i23) ab2,n=6.12;

(%o23) 327.19525505105713

(%i24) "**********"$

(%i25) " Die zweite Ableitung ist NULL, daher liegt ein Minimum vor "$

(%i26) "**********"$

(%i27) L(n):=c*n+x/(2*n)*p*i/100;

(%o27) $L(n) := c\ n + \frac{\frac{x}{2\ n}\ p\ i}{100}$

(%i28) ab:diff(L(n),n);

(%o28) $c - \frac{i\ p\ x}{200\ n^2}$

(%i29) solve(ab=0,n);

(%o29) $[n = - \frac{\sqrt{\frac{i\ p\ x}{c}}}{10\sqrt{2}} , n = \frac{\sqrt{\frac{i\ p\ x}{c}}}{10\sqrt{2}}]$

(%i30) %,numer;

(%o30) $[n = - 0.070710678118654766 \left(\frac{i\ p\ x}{c}\right)^{0.5} , n = 0.070710678118654766 \left(\frac{i\ p\ x}{c}\right)^{0.5}]$

(%i32) n:sqrt(p*x*i)/sqrt(200);

(%o32) $\frac{\sqrt{i\ p\ x}}{10\sqrt{2}}$

(%i34) Bestellmenge:x/n;

(%o34) $\frac{10\sqrt{2}\ x}{\sqrt{i\ p\ x}}$

```
(%i1)  "*************************************************"$
(%i2)  " Rechenregeln für Logarithmen                    "$
(%i3)  "*************************************************"$
(%i4)  Regel1(x,y):=log(x*y)=log(x)+log(y);
```

$$\text{(\%o4)}\quad \text{Regel1}(x\ ,\ y) := \log(x\ y) = \log(x) + \log(y)$$

```
(%i5)  Regel2(x,y):=log(x/y)=log(x)-log(y);
```

$$\text{(\%o5)}\quad \text{Regel2}(x\ ,\ y) := \log\left(\frac{x}{y}\right) = \log(x) - \log(y)$$

```
(%i6)  Regel3(x,y):=log(x^y)=y*log(x);
```

$$\text{(\%o6)}\quad \text{Regel3}(x\ ,\ y) := \log\left(x^{y}\right) = y\ \log(x)$$

```
(%i7)
```

Rechenregeln

```
(%i1)  g1:-14*y=-200+6*x;
```

$$(\%o1)\quad -14\,y = 6\,x - 200$$

```
(%i2)  g2:2*x+5*y=67;
```

$$(\%o2)\quad 5\,y + 2\,x = 67$$

```
(%i3)  A:coefmatrix([g1,g2],[x,y]);
```

$$(\%o3)\quad \begin{bmatrix} -6 & -14 \\ 2 & 5 \end{bmatrix}$$

```
(%i4)  b:matrix([-200], [67]);
```

$$(\%o4)\quad \begin{bmatrix} -200 \\ 67 \end{bmatrix}$$

```
(%i5)  Loesung:invert(A).b;
```

$$(\%o5)\quad \begin{bmatrix} 31 \\ 1 \end{bmatrix}$$

```
(%i6)  "******************************************************"$
(%i7)  g1:-33*x=153+51*y;
```

$$(\%o7)\quad -33\,x = 51\,y + 153$$

```
(%i8)  g2:(13*x-15*y)/2=-727/2;
```

$$(\%o8)\quad \frac{13\,x - 15\,y}{2} = -\frac{727}{2}$$

```
(%i10)  A:coefmatrix([g1,g2],[x,y]);
```

$$(\%o10)\quad \begin{bmatrix} -33 & -51 \\ \frac{13}{2} & -\frac{15}{2} \end{bmatrix}$$

```
(%i11)  b:matrix([153], [-727/2]);
```

$$(\%o11)\quad \begin{bmatrix} 153 \\ -\frac{727}{2} \end{bmatrix}$$

```
(%i12)  Loesung:invert(A).b;
```

$$(\%o12)\quad \begin{bmatrix} -34 \\ 19 \end{bmatrix}$$

```
(%i13)  "******************************************************"$
(%i14)  g1:3*x-4*y=77;
```

$$(\%o14)\quad 3\,x - 4\,y = 77$$

```
(%i15)  g2:317/5+16/15*y=x;
```

$$(\%o15)\quad \frac{16\,y}{15} + \frac{317}{5} = x$$

```
(%i16)  A:coefmatrix([g1,g2],[x,y]);
```

(%o16)
$$\begin{bmatrix} 3 & -4 \\ -1 & \frac{16}{15} \end{bmatrix}$$

(%i17)
```
b:matrix([77], [-317/15]);
```

(%o17)
$$\begin{bmatrix} 77 \\ -\frac{317}{15} \end{bmatrix}$$

(%i18)
```
Loesung:invert(A).b;
```

(%o18)
$$\begin{bmatrix} 3 \\ -17 \end{bmatrix}$$

(%i19)

```
(%i1)  "*****************************************************"$
(%i2)  " Umsatz der einzelnen Filialen                       "$
(%i3)  "*****************************************************"$
(%i4)  a:matrix([120], [234], [38]);
```

Anwendung der Matrizenrechnung in der Wirtschaft

$$(\%o4)\quad \begin{bmatrix} 120 \\ 234 \\ 38 \end{bmatrix}$$

```
(%i5)  b:matrix([20], [264], [39]);
```

$$(\%o5)\quad \begin{bmatrix} 20 \\ 264 \\ 39 \end{bmatrix}$$

```
(%i6)  c:matrix([220], [34], [68]);
```

$$(\%o6)\quad \begin{bmatrix} 220 \\ 34 \\ 68 \end{bmatrix}$$

```
(%i7)  d:matrix([123], [23], [383]);
```

$$(\%o7)\quad \begin{bmatrix} 123 \\ 23 \\ 383 \end{bmatrix}$$

```
(%i8)  Gesamtabsatz:a+b+c+d;
```

$$(\%o8)\quad \begin{bmatrix} 483 \\ 555 \\ 528 \end{bmatrix}$$

```
(%i9)  p:matrix([300], [120], [800]);
```

$$(\%o9)\quad \begin{bmatrix} 300 \\ 120 \\ 800 \end{bmatrix}$$

```
(%i10)  preis:transpose(p);
```

$$(\%o10)\quad \begin{bmatrix} 300 & 120 & 800 \end{bmatrix}$$

```
(%i11)  Umsatz_a:preis.a;
(%o11)   94480
(%i12)  Umsatz_b:preis.b;
(%o12)   68880
(%i13)  Umsatz_c:preis.c;
(%o13)   124480
(%i14)  Umsatz_d:preis.d;
```

```
(%o14)  346060
(%i15)  "*************************************************"$
(%i16)  " Abrechnung einer Möbelfirma mit 4 Filialen       "$
(%i17)  "*************************************************"$
(%i18)
```

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```

(%i1) `m:matrix([1,2,3], [-3,2,4], [2,-3,1]);`

(%o1)
$$\begin{bmatrix} 1 & 2 & 3 \\ -3 & 2 & 4 \\ 2 & -3 & 1 \end{bmatrix}$$

(%i2) `determinant(m);`

(%o2) 51

(%i3) `invert(m);`

(%o3)
$$\begin{bmatrix} \frac{14}{51} & -\frac{11}{51} & \frac{2}{51} \\ \frac{11}{51} & -\frac{5}{51} & -\frac{13}{51} \\ \frac{5}{51} & \frac{7}{51} & \frac{8}{51} \end{bmatrix}$$

(%i4) `determinant(invert(m));`

(%o4) $\frac{1}{51}$

(%i5)

```
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```

```
(%i1)  "*****************************************************"$
(%i2)  " Diagonalmatrix "$
(%i3)  "*****************************************************"$
(%i4)  a:diagmatrix(3,4);
```

im Unterricht der Sekundarstufe II haben wir die Diagonalmatrix noch nicht gebraucht

$$(\%o4)\quad \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

```
(%i5)  b:diagmatrix(4,1);
```

$$(\%o5)\quad \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

```
(%i6)  c:diagmatrix(5,1);
```

$$(\%o6)\quad \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

```
(%i7)  invert(a);
```

$$(\%o7)\quad \begin{bmatrix} \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & 0 \\ 0 & 0 & \frac{1}{4} \end{bmatrix}$$

```
(%i8)  invert(b);
```

$$(\%o8)\quad \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

```
(%i9)  invert(c);
```

$$(\%o9)\quad \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

(%i10)

```
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```

(%i1)
```
"*****************************************************"$
```

(%i2)
```
" Einfache Matrizenrechnung "$
```

(%i3)
```
"*****************************************************"$
```

(%i4)
```
A:matrix([1,1], [1,-1]);
```

(%o4)
$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

(%i5)
```
B:matrix([2,3], [-4,1]);
```

(%o5)
$$\begin{bmatrix} 2 & 3 \\ -4 & 1 \end{bmatrix}$$

(%i6)
```
"*****************************************************"$
```

(%i7)
```
C:A+B;
```

(%o7)
$$\begin{bmatrix} 3 & 4 \\ -3 & 0 \end{bmatrix}$$

(%i8)
```
D:A-B;
```

(%o8)
$$\begin{bmatrix} -1 & -2 \\ 5 & -2 \end{bmatrix}$$

(%i9)
```
E:A.B;
```

(%o9)
$$\begin{bmatrix} -2 & 4 \\ 6 & 2 \end{bmatrix}$$

(%i10)
```
F:invert(A);
```

(%o10)
$$\begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{bmatrix}$$

(%i11)
```
G:invert(B);
```

(%o11)
$$\begin{bmatrix} \frac{1}{14} & -\frac{3}{14} \\ \frac{2}{7} & \frac{1}{7} \end{bmatrix}$$

(%i12)
```
H:A.F;
```

(%o12)
$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

```
(%i13)  I:B.G;
```

(%o13)
$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

```
(%i14)
```

```
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(%i1)  "*****************************************************"$
(%i2)  " EINHEITSMATRIZEN "$
(%i3)  "*****************************************************"$
(%i4)  A:ident(2);
```

$$(\%o4)\quad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

```
(%i5)  B:ident(3);
```

$$(\%o5)\quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

```
(%i6)  C:ident(4);
```

$$(\%o6)\quad \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

```
(%i7)  D:ident(5);
```

$$(\%o7)\quad \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

```
(%i8)  "*****************************************************"$
(%i9)  X1:A+4*A;
```

$$(\%o9)\quad \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

```
(%i10)  X2:B-3* B;
```

$$(\%o10)\quad \begin{bmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

```
(%i11)  X4:C+10*C;
```

(%o11)
$$\begin{bmatrix} 11 & 0 & 0 & 0 \\ 0 & 11 & 0 & 0 \\ 0 & 0 & 11 & 0 \\ 0 & 0 & 0 & 11 \end{bmatrix}$$

(%i12) `X5:D+2*D;`

(%o12)
$$\begin{bmatrix} 3 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 3 \end{bmatrix}$$

(%i13)

```
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```

(%i1) `a:matrix([1,2,3], [-3,2,-4], [1,-2,8]);`

(%o1)
$$\begin{bmatrix} 1 & 2 & 3 \\ -3 & 2 & -4 \\ 1 & -2 & 8 \end{bmatrix}$$

(%i2) `invert(a);`

(%o2)
$$\begin{bmatrix} \frac{2}{15} & -\frac{11}{30} & -\frac{7}{30} \\ \frac{1}{3} & \frac{1}{12} & -\frac{1}{12} \\ \frac{1}{15} & \frac{1}{15} & \frac{2}{15} \end{bmatrix}$$

(%i3) `a.invert(a);`

(%o3)
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(%i4) `"*****************************************************"$`

(%i6) `" a) Matrixeingabe "$`

(%i7) `" b) inverse Matrix "$`

(%i10) `" c) Probe: Matrix . Inverse = Einheitsmatrix "$`

(%i11) `"*****************************************************"$`

(%i12)

```
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```

(%i1) `"*****************************************************"$`

(%i2) `" Inverse und Determinanten "$`

(%i3) `"*****************************************************"$`

(%i4) `A:matrix([1,0,0], [3,-2,4], [-2,3,-1]);`

(%o4)
$$\begin{bmatrix} 1 & 0 & 0 \\ 3 & -2 & 4 \\ -2 & 3 & -1 \end{bmatrix}$$

(%i5) `determinant(A);`

(%o5) -10

(%i6) `B:invert(A);`

(%o6)
$$\begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{2} & \frac{1}{10} & \frac{2}{5} \\ -\frac{1}{2} & \frac{3}{10} & \frac{1}{5} \end{bmatrix}$$

(%i7) `determinant(B);`

(%o7) $-\frac{1}{10}$

(%i8) `C:A.B;`

(%o8)
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(%i9) `determinant(C);`

(%o9) 1

(%i10) `"*****************************************************"$`

(%i11) `D:4*A;`

(%o11)
$$\begin{bmatrix} 4 & 0 & 0 \\ 12 & -8 & 16 \\ -8 & 12 & -4 \end{bmatrix}$$

(%i12) `E:invert(D);`

(%o12)
$$\begin{bmatrix} \frac{1}{4} & 0 & 0 \\ \frac{1}{8} & \frac{1}{40} & \frac{1}{10} \\ -\frac{1}{8} & \frac{3}{40} & \frac{1}{20} \end{bmatrix}$$

(%i13) `determinant(D);`

(%o13) -640

(%i14) `determinant(E);`

(%o14) $-\frac{1}{640}$

(%i15) `F:D.E;`

(%o15)
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(%i16) `determinant(F);`

(%o16) 1

(%i17)

```
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```

(%i1)
```
"*****************************************************"$
```

(%i2)
```
" Einheitsmatrix "$
```

(%i3)
```
"*****************************************************"$
```

(%i4)
```
A:matrix([1,2,3], [-4,2,-3], [0,8,-1]);
```

(%o4)
$$\begin{bmatrix} 1 & 2 & 3 \\ -4 & 2 & -3 \\ 0 & 8 & -1 \end{bmatrix}$$

(%i5)
```
B:invert(A);
```

(%o5)
$$\begin{bmatrix} -\frac{11}{41} & -\frac{13}{41} & \frac{6}{41} \\ \frac{2}{41} & \frac{1}{82} & \frac{9}{82} \\ \frac{16}{41} & \frac{4}{41} & -\frac{5}{41} \end{bmatrix}$$

(%i6)
```
C:A.B;
```

(%o6)
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(%i7)
```
"*****************************************************"$
```

(%i8)
```
D:4*A;
```

(%o8)
$$\begin{bmatrix} 4 & 8 & 12 \\ -16 & 8 & -12 \\ 0 & 32 & -4 \end{bmatrix}$$

(%i9)
```
E:invert(D);
```

(%o9)
$$\begin{bmatrix} -\frac{11}{164} & -\frac{13}{164} & \frac{3}{82} \\ \frac{1}{82} & \frac{1}{328} & \frac{9}{328} \\ \frac{4}{41} & \frac{1}{41} & -\frac{5}{164} \end{bmatrix}$$

(%i10)
```
F:D.E;
```

$$(\%o10)\quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

```
(%i11)  "**************************************************"$
```

```
(%i12)  G:A+4*D;
```

$$(\%o12)\quad \begin{bmatrix} 17 & 34 & 51 \\ -68 & 34 & -51 \\ 0 & 136 & -17 \end{bmatrix}$$

```
(%i13)  H:invert(G);
```

$$(\%o13)\quad \begin{bmatrix} -\frac{11}{697} & -\frac{13}{697} & \frac{6}{697} \\ \frac{2}{697} & \frac{1}{1394} & \frac{9}{1394} \\ \frac{16}{697} & \frac{4}{697} & -\frac{5}{697} \end{bmatrix}$$

```
(%i14)  I:G.H;
```

$$(\%o14)\quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

```
(%i15)  "**************************************************"$
(%i16)
```

```
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```

(%i1)
```
"******************************************************"$
```

(%i2)
```
" Matrizenalgebra "$
```

(%i3)
```
"******************************************************"$
```

(%i4)
```
A:matrix([1,2,3], [-4,2,-1], [2,-2,5]);
```

(%o4)
$$\begin{bmatrix} 1 & 2 & 3 \\ -4 & 2 & -1 \\ 2 & -2 & 5 \end{bmatrix}$$

(%i5)
```
B:transpose(A);
```

(%o5)
$$\begin{bmatrix} 1 & -4 & 2 \\ 2 & 2 & -2 \\ 3 & -1 & 5 \end{bmatrix}$$

(%i6)
```
C:A+B;
```

(%o6)
$$\begin{bmatrix} 2 & -2 & 5 \\ -2 & 4 & -3 \\ 5 & -3 & 10 \end{bmatrix}$$

(%i7)
```
D:A-B;
```

(%o7)
$$\begin{bmatrix} 0 & 6 & 1 \\ -6 & 0 & 1 \\ -1 & -1 & 0 \end{bmatrix}$$

(%i8)
```
"******************************************************"$
```

(%i9)
```
E:A.B;
```

(%o9)
$$\begin{bmatrix} 14 & -3 & 13 \\ -3 & 21 & -17 \\ 13 & -17 & 33 \end{bmatrix}$$

(%i10)
```
F:B.A;
```

(%o10)
$$\begin{bmatrix} 21 & -10 & 17 \\ -10 & 12 & -6 \\ 17 & -6 & 35 \end{bmatrix}$$

(%i11)
```
G:E-F;
```

$$(\%o11)\quad \begin{bmatrix} -7 & 7 & -4 \\ 7 & 9 & -11 \\ -4 & -11 & -2 \end{bmatrix}$$

```
(%i12)  "******************************************************"$
```

```
(%i13)  H:A.C;
```

$$(\%o13)\quad \begin{bmatrix} 13 & -3 & 29 \\ -17 & 19 & -36 \\ 33 & -27 & 66 \end{bmatrix}$$

```
(%i14)  I:C.A;
```

$$(\%o14)\quad \begin{bmatrix} 20 & -10 & 33 \\ -24 & 10 & -25 \\ 37 & -16 & 68 \end{bmatrix}$$

```
(%i15)  "******************************************************"$
(%i16)  J:A.D;
```

$$(\%o16)\quad \begin{bmatrix} -15 & 3 & 3 \\ -11 & -23 & -2 \\ 7 & 7 & 0 \end{bmatrix}$$

```
(%i17)  K:D.A;
```

$$(\%o17)\quad \begin{bmatrix} -22 & 10 & -1 \\ -4 & -14 & -13 \\ 3 & -4 & -2 \end{bmatrix}$$

```
(%i18)
```

```
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```

```
(%i1)  "*****************************************************"$
(%i2)  " Kommutativgesetz gilt nicht bei Matrizenmultiplikation "$
(%i3)  "*****************************************************"$
(%i4)  A:matrix([1,1,1], [1,1,1], [1,1,1]);
```

$$(\%o4)\quad \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

```
(%i5)  B:matrix([1,1,1], [2,2,2], [3,3,3]);
```

$$(\%o5)\quad \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix}$$

```
(%i6)  C:A.B;
```

$$(\%o6)\quad \begin{bmatrix} 6 & 6 & 6 \\ 6 & 6 & 6 \\ 6 & 6 & 6 \end{bmatrix}$$

```
(%i7)  D:B.A;
```

$$(\%o7)\quad \begin{bmatrix} 3 & 3 & 3 \\ 6 & 6 & 6 \\ 9 & 9 & 9 \end{bmatrix}$$

```
(%i8)  "*****************************************************"$
(%i9)  E:matrix([2,2,2], [3,3,3], [4,4,4]);
```

$$(\%o9)\quad \begin{bmatrix} 2 & 2 & 2 \\ 3 & 3 & 3 \\ 4 & 4 & 4 \end{bmatrix}$$

```
(%i10)  F:A.E;
```

$$(\%o10)\quad \begin{bmatrix} 9 & 9 & 9 \\ 9 & 9 & 9 \\ 9 & 9 & 9 \end{bmatrix}$$

```
(%i11)  G:E.A;
```

```
(%o11)  [ 6   6   6  ]
        [ 9   9   9  ]
        [ 12  12  12 ]
```

```
(%i13)  "**************************************************"$
```

```
(%i14)
```

```
(%i1)  "*************************************************"$
(%i2)  " Mehrstufiger Produktionsprozess                 "$
(%i3)  "*************************************************"$
(%i4)  A:matrix([0,2,0], [0,0,4], [5,1,0]);
```

das ist ein schönes Anwendungsbeispiel

$$(\%o4)\quad \begin{bmatrix} 0 & 2 & 0 \\ 0 & 0 & 4 \\ 5 & 1 & 0 \end{bmatrix}$$

```
(%i5)  B:matrix([2,3], [0,2], [1,5]);
```

$$(\%o5)\quad \begin{bmatrix} 2 & 3 \\ 0 & 2 \\ 1 & 5 \end{bmatrix}$$

```
(%i6)  "*************************************************"$
(%i7)  " A ist der Zusammenhang zwischen Rohstoffen und  "$
(%i8)  " Halbfabrikaten, B ist der Zusammenhang zwischen "$
(%i9)  " Halbfabrikaten und Endprodukt                   "$
(%i10)  "*************************************************"$
(%i11)  C:A.B;
```

$$(\%o11)\quad \begin{bmatrix} 0 & 4 \\ 4 & 20 \\ 10 & 17 \end{bmatrix}$$

```
(%i12)  "*************************************************"$
(%i13)  " Das ist der Zusammenhang zwischen Rohstoffen und "$
(%i14)  " Endprodukt                                      "$
(%i15)  "*************************************************"$
(%i16)
```

```
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```

(%i1)
```
"*****************************************************"$
```

(%i2)
```
" Transponierte Matrix "$
```

(%i3)
```
"*****************************************************"$
```

(%i4)
```
A:matrix([1,2,3], [-4,2,-3], [-2,4,1]);
```

(%o4)
$$\begin{bmatrix} 1 & 2 & 3 \\ -4 & 2 & -3 \\ -2 & 4 & 1 \end{bmatrix}$$

(%i5)
```
B:4*A;
```

(%o5)
$$\begin{bmatrix} 4 & 8 & 12 \\ -16 & 8 & -12 \\ -8 & 16 & 4 \end{bmatrix}$$

(%i6)
```
C:A+B;
```

(%o6)
$$\begin{bmatrix} 5 & 10 & 15 \\ -20 & 10 & -15 \\ -10 & 20 & 5 \end{bmatrix}$$

(%i7)
```
D:invert(A);
```

(%o7)
$$\begin{bmatrix} -7 & -5 & 6 \\ -5 & -\frac{7}{2} & \frac{9}{2} \\ 6 & 4 & -5 \end{bmatrix}$$

(%i8)
```
"*****************************************************"$
```

(%i9)
```
E:transpose(A);
```

(%o9)
$$\begin{bmatrix} 1 & -4 & -2 \\ 2 & 2 & 4 \\ 3 & -3 & 1 \end{bmatrix}$$

(%i10)
```
F:transpose(B);
```

(%o10)
$$\begin{bmatrix} 4 & -16 & -8 \\ 8 & 8 & 16 \\ 12 & -12 & 4 \end{bmatrix}$$

(%i11)
```
G:transpose(C);
```

$$(\%o11)\quad \begin{bmatrix} 5 & -20 & -10 \\ 10 & 10 & 20 \\ 15 & -15 & 5 \end{bmatrix}$$

```
(%i12)  H:transpose(D);
```

$$(\%o12)\quad \begin{bmatrix} -7 & -5 & 6 \\ -5 & -\frac{7}{2} & 4 \\ 6 & \frac{9}{2} & -5 \end{bmatrix}$$

```
(%i13)
```

```
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provides bug reporting information.
(%i1)  " Erzeugen eines zweidimensionalen Feldes"$
(%i2)  "*************************************************"$
(%i3)  h[i,j]:=i-j;
(%o3)   h     := i - j
         i , j
(%i4)  genmatrix(h,3,3);
                [ 0   - 1   - 2 ]
                [               ]
(%o4)           [ 1    0    - 1 ]
                [               ]
                [ 2    1     0  ]
(%i5)  kill(h);
(%o5)   DONE
(%i6)  h[i,j]:=i+j;
(%o6)   h     := i + j
         i , j
(%i7)  genmatrix(h,3,3);
                [ 2  3  4 ]
                [         ]
(%o7)           [ 3  4  5 ]
                [         ]
                [ 4  5  6 ]
(%i8)
```

```
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```

(%i1)

```
" Ein Auto fährt mit 100 km/h und wird durch eine Schnellbremsung mit a
= 6 m/s² zum Stillstand gebracht. Wie lange dauert die Schnellbremsung?
Wie groß ist der Bremsweg? "$
```

(%i2) `v[0]:100/3.6;`

(%o2) 27.777777777777779

ein Beispiel aus der Kinematik

(%i3) `a:6;`

(%o3) 6

(%i4) `v(t):=v[0]-a*t;`

(%o4) $v(t) := v_0 - a\,t$

(%i5) `integrate(v(t),t);`

(%o5) $27.777777777777779\,t - 3\,t^2$

(%i6) `s(t):=27.8*t-3*t^2;`

(%o6) $s(t) := 27.800000000000001\,t - 3\,t^2$

(%i7) `solve(v(t)=0,t);`

```
RAT replaced 27.77777777777778 by 250//9 = 27.77777777777778
```

(%o7) $[\,t = \frac{125}{27}\,]$

(%i8) `%,numer;`

(%o8) [t = 4.6296296296296298]

(%i9) `s(4.63);`

(%o9) 64.403300000000002

(%i10)

```
(%i1)  "*****************************************************"$
(%i2)  " Polynomdivision                                     "$
(%i3)  "*****************************************************"$
(%i4)  p1:(0.1*x^3-0.3*x^2+0.5*x-0.7);
(%o4)   0.10000000000000001 x^3 - 0.29999999999999999 x^2 + 0.5 x -
0.69999999999999996
(%i5)  p2:x-2.18;
(%o5)   x - 2.1800000000000002
(%i6)  divide(p1,p2);
RAT replaced -0.7 by -7//10 = -0.7
RAT replaced 0.5 by 1//2 = 0.5
RAT replaced -0.3 by -3//10 = -0.3
RAT replaced 0.1 by 1//10 = 0.1
RAT replaced -2.18 by -109//50 = -2.18
```

$$(\%o6)\quad \left[\frac{2500\,x^2 - 2050\,x + 8031}{25000},\ \frac{379}{1250000}\right]$$

```
(%i7)  %,numer;
```

$$(\%o7)\quad \left[4.0\ 10^{-5}\left(2500\,x^2 - 2050\,x + 8031\right),\ 3.032\ 10^{-4}\right]$$

```
(%i8)  expand(%);
(%o8)   [ 0.10000000000000001 x^2 - 0.082000000000000003 x +
0.32124000000000003 , 3.032 10^-4 ]
(%i9)
```

```
(%i1)  "*****************************************************"$
(%i2)  " Regressionsbeispiel                                 "$
(%i3)  "*****************************************************"$
(%i4)  x:[10,21,29,40,50];
(%o4)   [ 10 , 21 , 29 , 40 , 50 ]
(%i5)  y:2*x-1;
(%o5)   [ 19 , 41 , 57 , 79 , 99 ]
(%i7)  "*****************************************************"$
(%i8)  " Kontrolle der Daten mittels Regression              "$
(%i9)  "*****************************************************"$
(%i10)  s:[1,1,1,1,1];
(%o10)   [ 1 , 1 , 1 , 1 , 1 ]
(%i11)  load(vect);
(%o11)
C:/Programme/Maxima-5.9.1/share/maxima/5.9.1/share/vector/vect.mac
(%i12)  sx:x.s;
(%o12)   150
(%i13)  sy:y.s;
(%o13)   295
(%i15)  sxy:(x*y).s;
(%o15)   10814
(%i17)  sx2:(x^2).s;
(%o17)   5482
(%i18)  n:5;
(%o18)   5
(%i19)  "*****************************************************"$
(%i20)  g1:a*sx2+b*sx=sxy;
(%o20)   150 b + 5482 a = 10814
(%i21)  g2:a*sx+b*n=sy;
(%o21)   5 b + 150 a = 295
(%i22)  "*****************************************************"$
(%i23)  solve([g1,g2],[a,b]);
(%o23)   [ [ a = 2 , b = - 1 ] ]
(%i24)  "*****************************************************"$
(%i26)  y(x):=2*x+1;
```

Regression ist gut machbar

```
(%i1)  "*****************************************************"$
(%i2)  " Beispiel - Lineare Regression                       "$
(%i3)  "*****************************************************"$
(%i4)  x:[10,32,34,45,53,65];
(%o4)   [ 10 , 32 , 34 , 45 , 53 , 65 ]
(%i5)  y:10*x-28;
(%o5)   [ 72 , 292 , 312 , 422 , 502 , 622 ]
(%i6)  "*****************************************************"$
(%i7)  load(vect);
(%o7)
C:/Programme/Maxima-5.9.1/share/maxima/5.9.1/share/vector/vect.mac
(%i8)  s:[1,1,1,1,1,1];
(%o8)   [ 1 , 1 , 1 , 1 , 1 , 1 ]
(%i9)  n:6;
(%o9)   6
(%i10)  "*****************************************************"$
(%i11)  sx:x.s;
(%o11)   239
(%i12)  sy:y.s;
(%o12)   2222
(%i13)  sxy:(x*y).s;
(%o13)   106698
(%i14)  sx2:(x^2).s;
(%o14)   11339
(%i15)  "*****************************************************"$
(%i16)  g1:a*sx2+b*sx=sxy;
(%o16)   239 b + 11339 a = 106698
(%i17)  g2:a*sx+b*n=sy;
(%o17)   6 b + 239 a = 2222
(%i18)  "*****************************************************"$
(%i19)  solve([g1,g2],[a,b]);
(%o19)   [ [ a = 10 , b = - 28 ] ]
(%i21)  kill(all);
(%o0)   DONE
(%i1)  y=10*x-28;
(%o1)   y = 10 x - 28
(%i2)  "*****************************************************"$
(%i3)  " Ergebnis                                            "$
```

```
(%i1)  "*************************************************"$
(%i2)  " Quadratische Regression                         "$
(%i3)  "*************************************************"$
(%i4)
a*sum(x[i]^4,i,1,n)+b*sum(x[i]^3,i,1,n)+c*sum(x[i]^2,i,1,n)=sum(x[i]^2*y[
i],i,1,n);
```

$$(\%o4)\quad a\left(\sum_{i=1}^{n} x_i^{4}\right)+b\left(\sum_{i=1}^{n} x_i^{3}\right)+c\sum_{i=1}^{n} x_i^{2}=\sum_{i=1}^{n} x_i^{2}\,y_i$$

```
(%i6)
a*sum(x[i]^3,i,1,n)+b*sum(x[i]^2,i,1,n)+c*sum(x[i],i,1,n)=sum(x[i]*y[i],
i,1,n);
```

$$(\%o6)\quad a\left(\sum_{i=1}^{n} x_i^{3}\right)+b\left(\sum_{i=1}^{n} x_i^{2}\right)+c\sum_{i=1}^{n} x_i=\sum_{i=1}^{n} x_i\,y_i$$

```
(%i7)  a*sum(x[i]^2,i,1,n)+b*sum(x[i],i,1,n)+c*n=sum(y[i],i,1,n);
```

$$(\%o7)\quad c\,n+a\left(\sum_{i=1}^{n} x_i^{2}\right)+b\sum_{i=1}^{n} x_i=\sum_{i=1}^{n} y_i$$

```
(%i8)  "*************************************************"$
(%i9)  " das sind die Regressionsgleichungen             "$
(%i10)  "*************************************************"$
(%i11)  sx:15;
(%o11)   15
(%i12)  sy:871.8;
(%o12)   871.79999999999995
(%i13)  sx4:979;
(%o13)   979
(%i14)  sx3:225;
(%o14)   225
(%i15)  sx2:55;
(%o15)   55
(%i16)  sx2y:10862.7;
(%o16)   10862.700000000001
(%i17)  sxy:2824.9;
(%o17)   2824.9000000000001
(%i18)  "*************************************************"$
```

```
(%i19)  " Diese Summen wurden mit TK errechnet                "$
(%i20)  "*************************************************"$
(%i21)  g1:a*sx4+b*sx3+c*sx2=sx2y;
(%o21)   55 c + 225 b + 979 a = 10862.700000000001
(%i22)  g2:a*sx3+b*sx2+c*sx=sxy;
(%o22)   15 c + 55 b + 225 a = 2824.9000000000001
(%i23)  n:5;
(%o23)   5
(%i24)  g3:a*sx2+b*sx+c*n=sy;
(%o24)   5 c + 15 b + 55 a = 871.79999999999995
(%i25)  "*************************************************"$
(%i26)  solve([g1,g2,g3],[a,b,c]);
RAT replaced -10862.7 by -108627//10 = -10862.7
RAT replaced -2824.9 by -28249//10 = -2824.9
RAT replaced -871.8 by -4359//5 = -871.8
```

(%o26) $[\ [\ a = \frac{159}{140}\ ,\ b = \frac{1979}{140}\ ,\ c = \frac{5973}{50}\ ]\ ]$

```
(%i27)  %,numer;
(%o27)   [ [ a = 1.1357142857142857 , b = 14.135714285714286 , c =
119.45999999999999 ] ]
(%i28)  K=1.136*x^2+14.136*x+119.46;
(%o28)   K = 1.1360000000000001 x^2 + 14.135999999999999 x +
119.45999999999999
(%i29)  "*************************************************"$
(%i34)
```

```
(%i1)  "*****************************************************"$
(%i2)  " Berechnung der Summen mit Maxima                    "$
(%i3)  "*****************************************************"$
(%i4)  "x          y          x^4        x^3        x^2        x^2*y      x*y
          1      134,5          1          1          1      134,5      134,5
          2      152,1         16          8          4      608,4      304,2
          3        174         81         27          9       1566        522
          4      191,8        256         64         16     3068,8      767,2
          5      219,4        625        125         25       5485       1097
         15      871,8        979        225         55    10862,7     2824,9
"$
(%i5)  load(vect);
(%o5)
C:/Programme/Maxima-5.9.1/share/maxima/5.9.1/share/vector/vect.mac
(%i6)  x:[1,2,3,4,5];
(%o6)   [ 1 , 2 , 3 , 4 , 5 ]
(%i7)  y:[134.5,152.1,174,191.8,219.4];
(%o7)   [ 134.5 , 152.09999999999999 , 174 , 191.80000000000001 ,
219.40000000000001 ]
(%i8)  s:[1,1,1,1,1];
(%o8)   [ 1 , 1 , 1 , 1 , 1 ]
(%i9)  sx:s.x;
(%o9)   15
(%i10)  sy:s.y;
(%o10)   871.80000000000007
(%i11)  sx4:s.x^4;
(%o11)   979
(%i12)  sx3:s.x^3;
(%o12)   225
(%i13)  sx2:s.x^2;
(%o13)   55
(%i15)  xy:x*y;
(%o15)   [ 134.5 , 304.19999999999999 , 522 , 767.20000000000005 , 1097.0 ]
(%i18)  sxy:s.xy;
(%o18)   2824.9000000000001
(%i19)  x2y:x^2*y;
(%o19)   [ 134.5 , 608.39999999999998 , 1566 , 3068.8000000000002 , 5485.0 ]
(%i20)  sx2y:s.x2y;
(%o20)   10862.700000000001
(%i21)
```

```
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Dedicated to the memory of William Schelter.
This is a development version of Maxima. The function bug_report()
provides bug reporting information.
(%i1)  "*****************************************************"$
(%i2)  " Berechnung von Summen "$
(%i3)  "*****************************************************"$
(%i4)  sum(i^2,i,1,n);
```

$$(\%o4)\quad \sum_{i=1}^{n} i^2$$

```
(%i5)  %,simpsum;
```

$$(\%o5)\quad \frac{2\,n^3+3\,n^2+n}{6}$$

```
(%i6)  "*****************************************************"$
(%i7)  sum(i,i,1,n);
```

$$(\%o7)\quad \sum_{i=1}^{n} i$$

```
(%i8)  %,simpsum;
```

$$(\%o8)\quad \frac{n^2+n}{2}$$

```
(%i9)  "*****************************************************"$
(%i10)  sum(i^3,i,1,n);
```

$$(\%o10)\quad \sum_{i=1}^{n} i^3$$

```
(%i11)  %,simpsum;
```

$$(\%o11)\quad \frac{n^4+2\,n^3+n^2}{4}$$

```
(%i12)  "*****************************************************"$
(%i13)
```

```
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(%i1)  "*****************************************************"$
(%i2)  " Divergente Reihen "$
(%i3)  "*****************************************************"$
(%i4)  sum(1/x,x,1,10);
```

$$(\%o4)\quad \frac{7381}{2520}$$

```
(%i5)  1/1+1/2+1/3+1/4+1/5+1/6+1/7+1/8+1/9+1/10;
```

$$(\%o5)\quad \frac{7381}{2520}$$

```
(%i6)  f(n):=sum(1/x,x,1,10*n);
```

$$(\%o6)\quad f\left(n\right) := \mathrm{SUM}\left(\frac{1}{x}, x, 1, 10\,n\right)$$

```
(%i7)  makelist(f(n),n,1,5);
```

$$(\%o7)\quad \left[\frac{7381}{2520}, \frac{55835135}{15519504}, \frac{9304682830147}{2329089562800}, \frac{2078178381193813}{485721041551200}, \frac{1394323757722405496 0759}{3099044504245996706400}\right]$$

```
(%i8)  %,numer;
```

$$(\%o8)\quad [2.9289682539682538, 3.5977396571436819, 3.994987130920391, 4.2785430389363759, 4.4992053383294257]$$

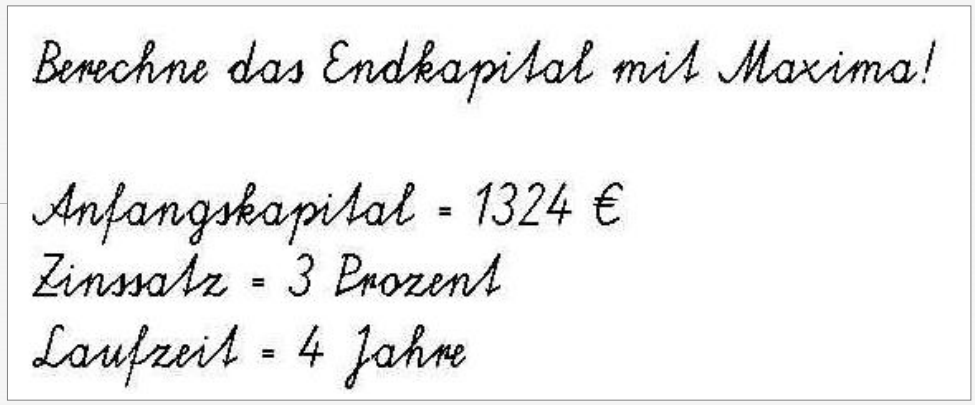
```
(%i9)  sum(1/x,x,1,INF);
```

$$(\%o9)\quad \sum_{x=1}^{\inf} \frac{1}{x}$$

```
(%i10)  %,simpsum;
(%o10)  inf
(%i11)
```

```
(%i1)  "Rekursive Berechnung von Fakultäten"$
(%i2)  x[0]:1;
(%o2)   1
(%i4)  f(n):=x[n+1]:x[n]*(n+1);
(%o4)   f(n) := x[n + 1] : x[n] (n + 1)
(%i5)  "*****************************************************"$
(%i6)  f(0);
(%o6)   1
(%i7)  f(1);
(%o7)   2
(%i8)  f(2);
(%o8)   6
(%i9)  f(3);
(%o9)   24
(%i10)  "*****************************************************"$
(%i11)  " usw.                                               "$
(%i12)  "*****************************************************"$
(%i13)
```

Rekursive Berechnung der Zinseszinsen



Aufzinsungsfaktor

Endkapital

```
(%i1)  "*****************************************************"$
(%i2)  " Rekursive Berechnung der Zinseszinsen               "$
(%i3)  "*****************************************************"$
(%i4)  K[0]:1324;
(%o4)   1324
(%i5)  r:1.03;
(%o5)   1.03
(%i6)  f(n):=K[n]:K[n-1]*r;
(%o6)   f(n) := K_n : K_(n - 1) r
(%i7)  f(1);
(%o7)   1363.72
(%i8)  f(2);
(%o8)   1404.6316000000002
(%i9)  f(3);
(%o9)   1446.7705480000002
(%i10)  f(4);
(%o10)   1490.1736644400003
(%i11)
```

```
(%i1)  "*****************************************************"$
(%i2)  " Statistische Kennzahlen                             "$
(%i3)  "*****************************************************"$
(%i4)  load(vect);
(%o4)
C:/Programme/Maxima-5.9.1/share/maxima/5.9.1/share/vector/vect.mac
(%i5)  x:[0,1,2,3,4];
(%o5)   [ 0 , 1 , 2 , 3 , 4 ]
(%i6)  h:3*x+10;
(%o6)   [ 10 , 13 , 16 , 19 , 22 ]
(%i8)  s:[1,1,1,1,1];
(%o8)   [ 1 , 1 , 1 , 1 , 1 ]
(%i9)  "*****************************************************"$
(%i10)  N:h.s;
(%o10)   80
(%i11)  p:h/N;
(%o11)   [1/8 , 13/80 , 1/5 , 19/80 , 11/40]
(%i12)  p.s;
(%o12)   1
(%i13)  "*****************************************************"$
(%i14)  " Summenprobe                                        "$
(%i15)  "*****************************************************"$
(%i16)  m:p.x;
(%o16)   19/8
(%i17)  "*****************************************************"$
(%i18)  " Erwartungswert                                     "$
(%i19)  "*****************************************************"$
(%i20)  v:p.((x-m)^2);
(%o20)   119/64
(%i21)  "*****************************************************"$
(%i22)  " Varianz                                            "$
(%i23)  "*****************************************************"$
(%i25)  st=sqrt(v);
(%o25)   st = sqrt(119)/8
(%i26)  "*****************************************************"$
```

```
(%i27)  " Streuung                                                    "$
(%i28)  "*************************************************************"$
(%i29)
```

```
(%i1)  "*****************************************************"$
(%i2)  " Eine einfache statistische Aufgabe                  "$
(%i3)  "*****************************************************"$
(%i4)  x:[0,1,2,3,4];
(%o4)   [ 0 , 1 , 2 , 3 , 4 ]
(%i5)  h:[30,12,8,3,1];
(%o5)   [ 30 , 12 , 8 , 3 , 1 ]
(%i6)  N:sum(h[i],i,1,5);
(%o6)   54
(%i7)  p:h/N;
```

$$(\%o7)\quad [\frac{5}{9},\frac{2}{9},\frac{4}{27},\frac{1}{18},\frac{1}{54}]$$

```
(%i8)  "*****************************************************"$
(%i10)  m:sum(x[i]*p[i],i,1,5);
```

$$(\%o10)\quad \frac{41}{54}$$

```
(%i11)  " Das ist der Erwartungswert                         "$
(%i12)  "*****************************************************"$
(%i13)  x1:(x-m)^2;
```

$$(\%o13)\quad [\frac{1681}{2916},\frac{169}{2916},\frac{4489}{2916},\frac{14641}{2916},\frac{30625}{2916}]$$

```
(%i15)  v:sum(x[i]*p[i],i,1,5);
```

$$(\%o15)\quad \frac{41}{54}$$

```
(%i16)  " Das ist die Varianz                                "$
(%i17)  "*****************************************************"$
(%i18)  s:sqrt(v);
```

$$(\%o18)\quad \frac{\sqrt{41}}{3\sqrt{6}}$$

```
(%i19)  " Das ist die Streuung                               "$
(%i20)  "*****************************************************"$
(%i21)
```

```
(%i1)  "*****************************************************"$
(%i2)  " Kennzahlen Schularbeitsergebnis                     "$
(%i3)  "*****************************************************"$
(%i4)  x:[1,2,3,4,5];
(%o4)   [ 1 , 2 , 3 , 4 , 5 ]
(%i5)  h:[1,3,10,3,1];
(%o5)   [ 1 , 3 , 10 , 3 , 1 ]
(%i6)  N:sum(h[i],i,1,5);
(%o6)   18
(%i7)  "*****************************************************"$
(%i8)  " 18 haben mitgeschrieben                             "$
(%i9)  "*****************************************************"$
(%i10)  p:h/N;
(%o10)   [1/18 , 1/6 , 5/9 , 1/6 , 1/18]
(%i11)  "*****************************************************"$
(%i12)  " Wahrscheinlichkeitsverteilung                      "$
(%i13)  "*****************************************************"$
(%i14)  m:sum(p[i]*x[i],i,1,5);
(%o14)   3
(%i15)  "*****************************************************"$
(%i16)  " Das der Erwartungswert                             "$
(%i17)  "*****************************************************"$
(%i18)  x1:(x-m)^2;
(%o18)   [ 4 , 1 , 0 , 1 , 4 ]
(%i19)  "*****************************************************"$
(%i20)  " Quadratische Abweichung vom Mittelwert             "$
(%i21)  "*****************************************************"$
(%i22)  v:sum(p[i]*x1[i],i,1,5);
(%o22)   7/9
(%i23)  "*****************************************************"$
(%i24)  " Das ist die Varianz                                "$
(%i25)  "*****************************************************"$
(%i26)  s:sqrt(v);
(%o26)   sqrt(7)/3
(%i27)  "*****************************************************"$
(%i30)  " Das ist die Streuung                               "$
```

```
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(%i1)  "*************************************************"$
(%i2)  " Faktorenzerlegung "$
(%i3)  "*************************************************"$
(%i4)  t1:x^2-8*x+15;
(%o4)   x^2 - 8 x + 15
(%i5)  t2:x^2-8*x+16;
(%o5)   x^2 - 8 x + 16
(%i6)  t3:x^2-8*x+17;
(%o6)   x^2 - 8 x + 17
(%i7)  "*************************************************"$
(%i8)  " Das sind die gegebenen Terme "$
(%i9)  "*************************************************"$
(%i10)  factor(t1);
(%o10)  (x - 5)(x - 3)
(%i11)  factor(t2);
(%o11)  (x - 4)^2
(%i12)  factor(t3);
(%o12)   x^2 - 8 x + 17
(%i13)  "*************************************************"$
(%i14)  " Es gibt noch ein die Funktion GFACTOR()"$
(%i15)  "*************************************************"$
(%i16)  gfactor(t1);
(%o16)  (x - 5)(x - 3)
(%i17)  gfactor(t2);
(%o17)  (x - 4)^2
(%i18)  gfactor(t3);
(%o18)  (x - %i - 4)(x + %i - 4)
(%i19)  "*************************************************"$
(%i20)  " GFACTOR() lässt auch komplexe Ergebnisse zu "$
(%i21)  "*************************************************"$
(%i22)
```

es ist praktisch, Objektbezeichner t1, t2 und t3 für die Terme zu verwenden

```
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provides bug reporting information.
(%i1)  "**************************************************"$
(%i2)  " Multiplikation von Binomem und darausfolgende Gleichungen "$
(%i3)  "**************************************************"$
(%i4)  t1:(x-4);
(%o4)   x - 4
(%i5)  t2:(x+4);
(%o5)   x + 4
(%i6)  t3:(x^2-1);
(%o6)   x^2 - 1
(%i7)  t4:(x^2+1);
(%o7)   x^2 + 1
(%i8)  "**************************************************"$
(%i9)  " Das sind die gegebenen Terme "$
(%i10)  "**************************************************"$
(%i11)  p1:t1*t2;
(%o11)   (x - 4)(x + 4)
(%i12)  expand(%);
(%o12)   x^2 - 16
(%i13)  solve(%=0,x);
(%o13)   [ x = - 4 , x = 4 ]
(%i14)  "**************************************************"$
(%i15)  p2:t1*t3;
(%o15)   (x - 4)(x^2 - 1)
(%i16)  expand(%);
(%o16)   x^3 - 4 x^2 - x + 4
(%i17)  solve(%=0,x);
(%o17)   [ x = 4 , x = - 1 , x = 1 ]
(%i18)  "**************************************************"$
(%i19)  p3:t1*t4;
(%o19)   (x - 4)(x^2 + 1)
(%i20)  expand(%);
```

es werden Terme erzeugt, die ganzzahlige Nullstellen haben müssen

```
(%i1)  "*****************************************************"$
(%i2)  " Trigonometrische Umformungen                        "$
(%i3)  "*****************************************************"$
(%i4)  t1:cos(x)*sin(x+y);
(%o4)  cos(x) sin(y + x)
(%i5)  trigexpand(t1);
(%o5)  cos(x) (cos(x) sin(y) + sin(x) cos(y))
(%i6)  trigreduce(t1);
                    sin(y + 2 x)   sin(y)
(%o6)               ------------ + ------
                          2          2
(%i7)  trigsimp(t1);
(%o7)  cos(x) sin(y + x)
(%i8)  "*****************************************************"$
(%i9)  t2:sin(x+y);
(%o9)  sin(y + x)
(%i10)  trigexpand(t2);
(%o10)  cos(x) sin(y) + sin(x) cos(y)
(%i11)  trigreduce(t2);
(%o11)  sin(y + x)
(%i12)  trigsimp(t2);
(%o12)  sin(y + x)
(%i13)  "*****************************************************"$
(%i14)  t3:sin(x+y)*cos(x+y);
(%o14)  cos(y + x) sin(y + x)
(%i15)  trigexpand(t3);
(%o15)  (cos(x) sin(y) + sin(x) cos(y)) (cos(x) cos(y) - sin(x) sin(y))
(%i16)  trigreduce(t3);
                    sin(2 y + 2 x)
(%o16)              --------------
                           2
(%i17)  trigsimp(t3);
(%o17)  cos(y + x) sin(y + x)
(%i18)  "*****************************************************"$
(%i19)  t4:(sin(x))^2+(cos(x))^2;
                      2         2
(%o19)  sin(x)^2 + cos(x)^2
(%i20)  trigexpand(t4);
                      2         2
(%o20)  sin(x)^2 + cos(x)^2
(%i21)  trigreduce(t4);
```

```
(%o21)  cos(2 x) + 1   1 - cos(2 x)
        ------------ + ------------
              2              2
(%i22)  trigsimp(t4);
(%o22)  1
(%i23)  "**************************************************"$
(%i24)
```

```
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(%i1)  "*****************************************************"$
(%i2)  " Trigonometrische Umformungen                        "$
(%i3)  "*****************************************************"$
(%i4)  a:cos(x)*sin(x+y);
(%o4)  cos(x) sin(y + x)
(%i5)  trigexpand(a);
(%o5)  cos(x) (cos(x) sin(y) + sin(x) cos(y))
(%i6)  trigreduce(%);
(%o6)  cos(x) sin(y + x)
(%i7)  trigreduce(a);
                    sin(y + 2 x)   sin(y)
(%o7)               ------------ + ------
                          2          2
(%i8)  "*****************************************************"$
(%i9)  b:sin(x)*cos(x+y);
(%o9)  sin(x) cos(y + x)
(%i10)  trigexpand(b);
(%o10)  sin(x) (cos(x) cos(y) - sin(x) sin(y))
(%i11)  trigreduce(%);
(%o11)  sin(x) cos(y + x)
(%i12)  trigreduce(%);
                    sin(y + 2 x)   sin(y)
(%o12)              ------------ - ------
                          2          2
(%i13)  "*****************************************************"$
(%i14)  c:sin(x)^2+cos(x)^2;
                       2       2
(%o14)  sin(x)^2 + cos(x)^2
(%i15)  trigexpand(c);
                       2       2
(%o15)  sin(x)^2 + cos(x)^2
(%i16)  trigreduce(%);
                    cos(2 x) + 1   1 - cos(2 x)
(%o16)              ------------ + ------------
                          2              2
(%i17)  trigreduce(c);
```

```
(%o17)  cos(2 x) + 1   1 - cos(2 x)
        ------------ + ------------
              2              2
(%i18)  expand(%);
(%o18)  1
(%i19)  "**************************************************"$
(%i20)
```

Die Vektorrechung ist auch in der Wirtschaftsmathematik nützlich

```
// Package Vektorrechnung aufrufen
// Erwartungswert mit Skalarprodukt berechnen
// ==========================================

(%i1) load(vect);
(%o1) C:/Programme/Maxima-5.9.0/share/maxima/5.9.1/share/vector/vect.mac

// x = Anzahl der Verkehrsunfälle pro Woche
// ========================================

(%i2) x:[0,1,2,3,4];
(%o2)                          [0, 1, 2, 3, 4]

// h = Häufigkeitsverteilung (absolut)
// ===================================

(%i3) h:[30,12,8,0,2];
(%o3)                         [30, 12, 8, 0, 2]

// summierender Vektor
// ===================

(%i4) s:[1,1,1,1,1];
(%o4)                          [1, 1, 1, 1, 1]

// Summe der Häufigkeiten
// ======================

(%i5) N:h.s;
(%o5)                                  52

// p = relative Häufigkeiten (Wahrscheinlichkeiten)
// ================================================

(%i6) p:h/N;
                           15  3   2      1
(%o6)                     [--, --, --, 0, --]
                           26  13  13     26

// Berechnung des Erwartungswerts
// ==============================

(%i8) E:p.x;
                                   9
(%o8)                              --
                                   13
(%i9)
```

```
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```

(%i1) `"*****************************************************"$`

(%i2) `" Binomialverteilung "$`

(%i3) `"*****************************************************"$`

(%i4) `c(n,k):=n!/k!/(n-k)!;`

(%o4) $$c(n,k) := \frac{\frac{n!}{k!}}{(n-k)!}$$

es geht auch

binom(n,k)

oder

binomial(n,k)

(%i5) `"*****************************************************"$`

(%i8) `" benutzerdefinierte Binomialkoeffizienten "$`

(%i9) `"*****************************************************"$`

(%i10) `B(n,k,p):=c(n,k)*p^k*(1-p)^(n-k);`

(%o10) $$B(n,k,p) := c(n,k)\, p^k\, (1-p)^{n-k}$$

(%i11) `n:4;`

(%o11) 4

(%i12) `p:1/6;`

(%o12) $\frac{1}{6}$

(%i13) `makelist(B(n,k,p),k,0,n);`

(%o13) $$\left[\frac{625}{1296}, \frac{125}{324}, \frac{25}{216}, \frac{5}{324}, \frac{1}{1296}\right]$$

(%i14) `"*****************************************************"$`

(%i15) `" Das ist die Binomialverteilung "$`

(%i16) `"*****************************************************"$`

(%i17) `sum(B(n,k,p),k,0,n);`

(%o17) 1

(%i18) `"*****************************************************"$`

(%i19) `" Die Summenprobe ergibt 1 "$`

(%i20) `"*****************************************************"$`

(%i21)

```
(%i1)  "*****************************************************"$
(%i2)  " Zufallsgenerator                                    "$
(%i3)  "*****************************************************"$
(%i4)  makelist(random(i)+1,i,1,6);
(%o4)   [ 1 , 2 , 1 , 2 , 1 , 2 ]
(%i5)  makelist(random(i)+1,i,1,6);
(%o5)   [ 1 , 1 , 3 , 1 , 4 , 3 ]
(%i6)  makelist(random(i)+1,i,1,6);
(%o6)   [ 1 , 1 , 2 , 4 , 3 , 5 ]
(%i7)  makelist(random(i)+1,i,1,6);
(%o7)   [ 1 , 2 , 1 , 2 , 4 , 6 ]
(%i8)  makelist(random(i)+1,i,1,6);
(%o8)   [ 1 , 2 , 1 , 4 , 4 , 2 ]
(%i9)  makelist(random(i)+1,i,1,6);
(%o9)   [ 1 , 2 , 3 , 4 , 1 , 5 ]
(%i10)  makelist(random(i)+1,i,1,6);
(%o10)   [ 1 , 1 , 2 , 3 , 4 , 5 ]
(%i11)  makelist(random(i)+1,i,1,6);
(%o11)   [ 1 , 2 , 2 , 3 , 3 , 2 ]
(%i12)  "*****************************************************"$
(%i13)  " Simulation 6-mal würfeln                           "$
(%i14)  "*****************************************************"$
(%i15)
```

Wir simulieren mehrmals 6 Ergebnisse eines Würfelexperiments

```
wxMaxima 0.6.4 http://wxmaxima.sourceforge.net
Maxima 5.9.1 http://maxima.sourceforge.net
Using Lisp Kyoto Common Lisp GCL 2.6.5 (aka GCL)
Distributed under the GNU Public License. See the file COPYING.
Dedicated to the memory of William Schelter.
This is a development version of Maxima. The function bug_report()
provides bug reporting information.
```

```
(%i1)  "*****************************************************"$
(%i2)  " Zweidimensionale Felder                               "$
(%i3)  "*****************************************************"$
(%i4)  h[i,j]:=i+j;
(%o4)  h_i,j := i + j
(%i5)  g[i,j]:=i-j;
(%o5)  g_i,j := i - j
(%i6)  f[i,j]:=i*j;
(%o6)  f_i,j := i j
(%i7)  e[i,j]:=i/j;
(%o7)  e_i,j := i/j
(%i8)  "*****************************************************"$
(%i9)  A:genmatrix(h,3,3);
```

Zweidimensionale Felder sind in der Softwareerstellung insbesondere im Bereich der Wirtschaftsinformatik nützlich.

$$(\%o9)\quad \begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix}$$

```
(%i10) B:genmatrix(g,3,3);
```

$$(\%o10)\quad \begin{bmatrix} 0 & -1 & -2 \\ 1 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix}$$

```
(%i11) C:genmatrix(f,3,3);
```

$$(\%o11)\quad \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$$

```
(%i12) D:genmatrix(e,3,3);
```

$$(\%o12)\quad \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} \\ 2 & 1 & \frac{2}{3} \\ 3 & \frac{3}{2} & 1 \end{bmatrix}$$

```
(%i1)  "*************************************************"$
(%i2)  " Listen von Fakultäten                           "$
(%i3)  "*************************************************"$
(%i4)  f(n):=makelist(i!,i,0,n);
(%o4)   f(n) := MAKELIST(i ! , i , 0 , n)
(%i5)  "*************************************************"$
(%i6)  for n:0 thru 7 do display(n,f(n));
n = 0 f(0) = [ 1 ]
n = 1 f(1) = [ 1 , 1 ]
n = 2 f(2) = [ 1 , 1 , 2 ]
n = 3 f(3) = [ 1 , 1 , 2 , 6 ]
n = 4 f(4) = [ 1 , 1 , 2 , 6 , 24 ]
n = 5 f(5) = [ 1 , 1 , 2 , 6 , 24 , 120 ]
n = 6 f(6) = [ 1 , 1 , 2 , 6 , 24 , 120 , 720 ]
n = 7 f(7) = [ 1 , 1 , 2 , 6 , 24 , 120 , 720 , 5040 ]
(%o6)   DONE
(%i7)
```

```
(%i1)  "*************************************************"$
(%i2)  " Pascaldreieck                                   "$
(%i3)  "*************************************************"$
(%i4)  display2d:false;
(%o4)  FALSE
(%i5)  c(n,k):=n!/(k!*(n-k)!);
(%o5)  c(n,k):=n!/(k!*(n-k)!)
(%i6)  display2d:true;
(%o6)   true
(%i7)  c(n,k);
```

das schaltet die zweidimensionale Anzeige aus

das schaltet die zweidimensionale Anzeige wieder ein (das ist sinnvollerweise der Standardwert)

$$(\%o7)\quad \frac{n!}{k!\,(n-k)!}$$

```
(%i8)  "*************************************************"$
(%i9)  display2d:false;
(%o9)  FALSE
(%i10)  for n:0 thru 5 do for k:0 thru n do disp(c(n,k));
1
1
1
1
2
1
1
3
3
1
1
4
6
4
1
1
5
10
10
5
1
(%o10)  DONE
(%i11)  for n:0 thru 5 do for k:0 thru n do display(c(n,k));
c(0,0)  =  1
c(1,0)  =  1
c(1,1)  =  1
c(2,0)  =  1
```

für n=2

für n=4

```
c(2,1) = 2
c(2,2) = 1
c(3,0) = 1
c(3,1) = 3
c(3,2) = 3
c(3,3) = 1
c(4,0) = 1
c(4,1) = 4
c(4,2) = 6
c(4,3) = 4
c(4,4) = 1
c(5,0) = 1
c(5,1) = 5
c(5,2) = 10
c(5,3) = 10
c(5,4) = 5
c(5,5) = 1
(%o11) DONE
(%i12)
```

Diese Übungen wurden im Rahmen eines MNI-Projektes an der Bundeshandelsakademie und Bundeshandelsschule Tamsweg für den Gebrauch mit der Lernplattform Classserver von Microsoft erstellt.

Die verwendete Software stammt von:

http://maxima.sourceforge.net mit Zusatz
http://wxmaxima.sourceforge.net

Dokumentiert sind diese Beispiele auf

http://www.hit4u.at/maxima

Johann Weilharter

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